Pressure in an active fluid

"Pressure and Phase Equilibria in Interacting Active Brownian Spheres"
A. P. Solon, J. Stenhammar, R. Wittkowski, M. Kardar, Y. Kafri, M. E. Cates, J. Tailleur
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Recommended with commentary by Aparna Baskaran, Brandeis University

Introduction : The advent of quantitative biology has brought into the purview of condensed matter physicists a new class of inherently out of equilibrium materials that are driven at the microscale, rather than by body forces such as external fields or boundary forces such as shear. Such materials have been termed "active materials". For example, the cytoskeleton of a eukaryotic cell can be thought of as a polymer gel that is driven out of equilibrium by the activity of motor proteins and the growth, remodeling and destruction of the constituent polymer filaments. The cell uses biochemical regulation to modulate the mechanical properties and activity to use this material to accomplish diverse tasks including motility, surface adhesion and mechanosensing. While most systems of biological origin are complex, a simple realization of an active material is a collection of self-propelled colloidal spheres or disks. In the recent years this system has emerged as the "Ising model" for developing the theoretical framework applicable to this class of condensed matter systems
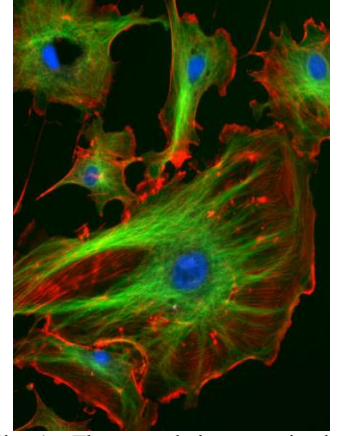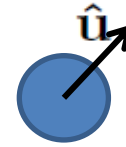

Fig 1: The cytoskeleton stained for actin (red) and microtubules (green) (image courtesy Wikipedia)

Context and Overview : For a conventional fluid, whether in equilibrium or driven, pressure is a quantity that is well defined and is given by the equation of state that one can calculate within the framework of equilibrium statistical mechanics. So the question that has been asked and answered in the last few months is : What is pressure in an active fluid? This question comes with two challenges. First we do not have an analog of equilibrium statistical mechanics for inherently driven systems such as active materials. Second, we cannot treat activity perturbatively and get results of universal validity because by definition the material is no longer active to zeroth order. In this commentary, three theoretical works that have contributed to our nascent understanding of pressure in active fluids are discussed, with the highlighted paper being the latest among these works.

$$\partial_t \mathbf{r} = v_0 \hat{\mathbf{u}}$$

$$\partial_t \theta = \eta^R$$

$$\langle v_i(t) v_j(t') \rangle = \frac{v_0^2}{D_R} \delta_{ij} e^{-D_R|t-t'|}$$

Fig 2: The "ising model" for an active fluid. The particle moves along one direction of this body axis. This direction diffuses in space. The velocity autocorrelation is that of a persistent random walk of persistence length v₀/Dr

Results and Discussion: The first of these papers [1] showed that a mechanical definition of bulk pressure using an Irving-Kirkwood expression [4] for a stress tensor makes sense and agrees with a measured wall pressure provided that one included a one-particle term (in bulk!) that arises purely due to the self-propulsion of the active colloids.

The second of these papers [2] uses a one-particle term in the bulk pressure exactly as in [1] which they call the "swim pressure", together with a phenomenological closure for the interaction pressure to get an explicit equation of state for the system. Further, they use the equilibrium relationship between chemical potential and pressure to construct an active free energy.

In the highlighted paper [3], the authors use the framework of non-equilibrium statistical mechanics to construct the pressure in the active system, starting from the microscopic Langevin equation, in terms of bulk correlators. They find a one-particle term as in the previous works which they now call "indirect pressure'. Further, using an approximate closed form expression for this pressure they study the phase behavior of an active fluid and show that the pressure is indeed equal in coexisting phases even though a naïve Maxwell construction fails to predict the densities of coexisting phases. They also place their work robustly in context with the papers discussed above.

These papers are a first step by the community to figure out a theoretical paradigm that will take the place of thermodynamics for these inherently out of equilibrium materials. We theorists in this field are at the precipice of what I am sure will prove exciting times for us.

References :

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