Collective Dynamics of Dividing Chemotactic Cells
Authors: Anatolij Gelimson and Ramin Golestanian,
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Recommended with a commentary by Mehran Kardar, MIT

To describe the collective behavior of a population of cells, the above paper introduces a coarse-grained model (for the cell density $\rho(\mathbf{x}, t)$), incorporating only two central processes, *population change through reproduction and death*, and *chemotaxis (movement in response to chemical signals)*:

- The former is included through the widely studied Fisher equation [1]

$$\partial_t \rho = \theta \rho \left(1 - \frac{\rho}{\rho_m}\right) + D\nabla^2 \rho + \eta(\mathbf{x}, t). \tag{1}$$

Cells grow at rate θ up to a maximum density ρ_m; the stochastic nature of cell growth leads to the "diffusion" term D, and the "white" noise $\eta(\mathbf{x}, t)$.

- Chemotaxis is described through a concentration $c(\mathbf{x}, t)$, produced by the cells as source; $\nabla^2 c = \rho$, leading to $c = \nabla^{-2}\rho$, assuming instantaneous dispersion. There is then a current of cells moving towards regions of high concentration gradient, $\mathbf{J} \propto -\rho\nabla c$.

Putting the two processes together, and adapting their notation $\nu_2 = 2\theta/\rho_m$, Gelimson and Golestanian arrive at a non-local generalization of the Fisher equation

$$\partial_t \rho = D\nabla^2 \rho + \theta\rho - \nu_1 \nabla \cdot \left[\rho \nabla \left(\frac{1}{\nabla^2}\right) \rho\right] - \frac{\nu_2}{2}\rho^2 + \eta \quad . \tag{2}$$

The two, equally relevant, non-linearities make this a formidable equation, yet undaunted the authors proceed to study it via dynamic renormalization group (RG). They are rewarded by finding a stable non-trivial fixed point (at finite ν_1 and ν_2) with a large basin of attraction (B_1). A sharp boundary (passing through an unstable fixed point) separates this basin (B_1) from another (B_2) in which ν_2 diverges.

From the perspective of formal RG, this is a highly interesting structure; its implications for collective cell behavior (albeit suggestive) are possibly even more striking. The authors interpret basin B_1 as describing a balance between growth and chemotaxis resulting in a tissue with well defined density, whereas in basin B_2 chemotaxis becomes irrelevant causing unregulated growth, reminiscent of tumor metastasis. Two fundamental processes governing collective cell behavior are certainly included (at coarse-grained level) in Eq. (2). Is it possible that dynamic RG analysis is in fact pointing to a non-trivial result in biology?

References

[1] R.A. Fisher, Ann. Eugen. London **7**, 355 (1937).