Lattice Kirigami

Algorithmic lattice kirigami: a route to pluripotent materials
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Recommended with a commentary by Mark Bowick, Soft Matter Program, Syracuse University

Origami - the Japanese art of paper folding - enters physics, mathematics materials science in many ways [1]. It can be viewed as a discrete version of the Helfrich energy [2] for the bending of two-dimensional membranes in three-dimensional Euclidean space, which is in turn equivalent to the Willmore functional in mathematics [3] measuring how far 2D manifolds deviate in shape from the round two-sphere. Some time ago Di Francesco and Guitter [4] studied the"Ising model" of folding of a triangular lattice in which folds could either be no-folds or 180-degree folds. That is, two triangular plaquettes which share an edge are either co-planar or folded completely on top of each other. Folding configurations correspond to an 11-vertex lattice model and can be mapped to the 3-coloring of the edges of the lattice - a model solved exactly by Baxter in 1970. From this one can compute the entropy of folding and also show that folding is global in the sense that there are no local folding degrees of freedom, only local unfolding - a fact quite clear when you fold paper. Origami experts also know that many folds are complex and cooperative and must be executed collectively rather than sequentially. All this makes origami a rich but limited technique for creating interesting 3D structures from a planar sheet, whether it is engineered or self-assembled.

Much greater freedom - and even simplicity - can be achieved by allowing the topology of the sheet to change, in particular by allowing cuts in the sheet together with folds. This is also a traditional Japanese art - *kirigami* - from "kiru" to cut and "kami" paper. In a lovely paper Sussman et al. develop an algorithm for forming a target curved surface (3D shape) from specific cuts and folds on a flat sheet.

A central kirigami element in the Sussman et al. construction is the dislocation or 5-7 pair on a triangular lattice, a familiar defect in materials science and in the theory of 2D melting. Each pair has four allowed folded configurations which become building blocks for stepped surfaces. Arrays of dislocations folded in various ways can then approximate a wide variety of 3D shapes. Design is aided by developing a simple graphical representation for the basic stepped surface building blocks.

One limitation of this first algorithm noted by the authors is that every desired target surface requires an entirely new pattern of cuts to be specified before subsequent folding into 3D. The authors then develop a more general

algorithm which uses "sixons" - the removal of a complete hexagon followed by reconnecting the cut edges of the remaining surface in one of several ways. The sixons are then distributed with their centers on a triangular lattice. This leads to a stepped surface with triangular plateaus which can approximate a target surface provided the gradients on the surface are not too large. Local excitations of a given structure lead to new surfaces all arising from the same basic blueprint.

The principles at work here are scale independent and thus apply from the nano to the macro-scale. Hard folds along creases are rather hard to make in general but cutting is easy via lithography. Many wonderful 2D materials such as graphene and molybdenum disulphide are available as raw materials for lattice kirigami surgery. The electronic properties of graphene are largely insensitive to changes in its shape so that flexible electronics from lattice kirigami should not be far away and would certainly be a cut above planar graphene.

References

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