Composite fermions meet Dirac

I. *Is the composite fermion a Dirac particle?*
Dam Thanh Son, Phys. Rev. X **5**, 031027 (2015); arXiv:1502.03446

II. *Dual Dirac liquid on the surface of the electron topological insulator*
Chong Wang and T. Senthil, arXiv:1505.05141

III. *Particle-vortex duality of 2D Dirac fermion from electric-magnetic duality of 3D topological insulators*
Max A. Metlitski and Ashvin Vishwanath, arXiv:1505.05142

IV. *Half-filled Landau level, topological insulator surfaces, and 3D quantum spin liquids*
Chong Wang and T. Senthil, arXiv:1507.08290

V. *The half-filled Landau level: the case for Dirac composite fermions*
Scott D. Geraedts, Michael P. Zaletel, Roger S. K. Mong, Max A. Metlitski, Ashvin Vishwanath, and Olexei I. Motrunich, arXiv:1508.04140

Recommended with a commentary by Jason Alicea, Caltech

At first glance, 3D topological insulator surfaces and the half-filled Landau level of a 2D electron gas appear to realize wholly disparate phenomena. Topological insulators typically arise in weakly correlated, time-reversal-invariant systems and support a single Dirac cone at their boundary. The half-filled Landau level, by contrast, is a venue for exotic strongly correlated phases unveiled at high magnetic fields. The papers highlighted here deliver profound new insights into these platforms that both resolve long-standing puzzles and expose intimate connections between the two.

Historically, the half-filled Landau level has been profitably attacked by mapping the electron to a composite fermion bound to two statistical flux quanta that, on average, exactly cancel the external magnetic flux. Composite fermions therefore experience zero field and form a 'composite Fermi liquid' predicted in classic work by Halperin, Lee, and Read (HLR) [1]. In a standard 2D electron gas, e.g., GaAs quantum wells, the underlying composite Fermi sea is believed to be stable in the lowest Landau level but unstable to pairing in the second Landau level—giving way to the non-Abelian Moore-Read state [2]. While many aspects of the HLR theory are borne out experimentally, a nagging issue has long persisted: The half-filled Landau level exhibits an exact particle-hole symmetry (neglecting Landau-level mixing) broken explicitly in the conventional composite-fermion flux-attachment description. This is 'exhibit A' in the case for a revised picture of the composite Fermi liquid.

For 'exhibit B' we note that earlier works hint at a deep relationship between the half-filled Landau level and topological insulators that is certainly obscured in traditional descriptions of the former. Consider the following circumstantial evidence. A single Dirac cone reminiscent of the topological insulator surface was discussed long ago in the context of the integer quantum Hall plateau transition [3]. Much more recently several groups constructed symmetric, fully gapped surfaces of a 3D topological insulator strikingly similar to the Moore-Read phase [4]; see the Oct. 2013 commentary by Ady Stern. Mross et al. later introduced 'composite Dirac liquid' topological-insulator surface states in which emergent electrically neutral fermions form a single Dirac cone. The neutral Dirac sea is analogous to the half-filled Landau level's composite Fermi sea [5], and when paired yields precisely the gapped Moore-Read-like phases identified previously. These tantalizing similarities are far from purely incidental.

In a remarkable stroke of intuition, Son boldly postulated that the composite fermion ψ in the half-filled Landau level *is* a Dirac particle and proposed the following alternative field theory for a composite Fermi liquid:

$$\mathcal{L} = \bar{\psi}\gamma^{\mu}(i\partial_{\mu} - a_{\mu})\psi + \frac{1}{4\pi}\epsilon_{\mu\nu\lambda}A_{\mu}\partial_{\nu}a_{\lambda} + \cdots. \tag{1}$$

Here a_μ is a gauge field whose flux encodes electrical currents, while A_μ denotes the external electromagnetic vector potential. Importantly, the Dirac fermions are doped away from neutrality and thus exhibit the familiar composite Fermi surface. In fact Son's reformulation recovers the essential phenomenology of the HLR theory—but in a manifestly particle-hole symmetric framework! More precisely, particle-hole symmetry was conjectured to transform the Dirac composite fermion ψ in the same way that time-reversal transforms Dirac electrons on a topological insulator surface. A nice consistency check is that violating particle-hole by adding a 'large' mass term for ψ allows one to recover precisely HLR's construction.

Follow-up work placed this theory on firmer footing. Wang and Senthil [paper IV] paint an appealing picture of the composite fermion as a dipolar bound state of $e/2$ and $-e/2$ charge/vortex composites (see also [6]). This bound state forms a Kramers doublet under particle-hole and exhibits an analogue of spin-momentum locking; together, these properties make Son's Dirac theory extremely natural. Extensive DMRG simulations by Geraedts et al. further establish compelling evidence that the (projected) half-filled lowest Landau level with long-range Coulomb interactions indeed realizes such a Dirac-like composite Fermi liquid. General arguments based on symmetry and Fermi-surface counting bolster this conclusion. Strikingly, they also find that certain correlation functions do *not* exhibit $2k_F$ singularities that would otherwise be present in the usual HLR description—implying suppression of perfect backscattering off of particle-hole-symmetric impurities. This feature has a similar origin to the famous suppression of backscattering from time-reversal-invariant disorder in a topological insulator surface, and is a telltale signature of Dirac fermions.

Ties between the half-filled Landau level and topological insulators run deeper still. Two independent works—Wang and Senthil [paper II], Metlitski and Vishwanath—show that the very same field theory in Eq. (1) just argued to describe the composite Fermi liquid *also* provides a dual description of the topological insulator surface! In the latter context the 'composite fermion' ψ represents a double-strength vortex in the electron fluid, and under physical time-reversal symmetry gets particle-hole conjugated. (Note the complementary role played by time-reversal and particle-hole in the two platforms.) A great virtue of dualities is that physics difficult to describe in the original degrees of freedom often becomes immediately transparent in a dual framework. This case is no exception: for instance, one of the Moore-Read-like symmetric gapped surface phases noted above corresponds simply to a paired state of dual fermions. Evidently these are the 'right' variables for capturing universal properties of the strongly interacting surface.

Numerous other gems abound in this collection of papers. Time-reversal-invariant 3D topological superconductors and spin liquids are also intricately woven into the storyline, yielding yet other fascinating aspects of the problem. A confluence of ideas to this degree is rare in physics and naturally spotlights many interesting questions for future research. For instance, can one construct a trial composite Fermi liquid wavefunction that makes particle-hole symmetry and the Dirac structure explicit? Experimentally, particle-hole symmetry will invariably be broken to some degree due to Landau level mixing and disorder. Can experiments nevertheless resolve signatures of (perhaps nearly massless) Dirac composite fermions? More formally, what can we learn about the low-energy behavior of Eq. (1)—a strongly interacting field theory—by exploiting the duality to the usual topological-insulator surface theory? Revisiting such problems with the fresh perspective surveyed here is likely to bear many more interesting results.

[1] B. I. Halperin, Patrick A. Lee, and Nicholas Read, Phys. Rev. B **47**, 7312 (1993)

[2] G. Moore and N. Read, Nuclear Physics B **360**, 362 (1991)

[3] A. W. W. Ludwig, M. P. A. Fisher, R. Shankar, and G. Grinstein, Phys. Rev. B **50**, 7526 (1994)

[4] P. Bonderson, C. Nayak, and X.-L. Qi, J. Stat. Mech. (2013) P09016; C. Wang, A. C. Potter, and T. Senthil, Phys. Rev. B **88**, 115137 (2013); X. Chen, L. Fidkowski, and A. Vishwanath, Phys. Rev. B **89**, 165132 (2014); M. A. Metlitski, C. L. Kane, and M. P. A. Fisher, arXiv:1306.3286

[5] D. F. Mross, A. Essin, and J. Alicea, Phys. Rev. X **5**, 011011 (2015)

[6] G. Murthy and R. Shankar, arXiv:1508.06974