Closing the loopholes on Bell's theorem

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From the earliest days of quantum mechanics (hereafter QM) it has been appreciated that the conceptual framework of quantum mechanics is very different from as that embodied in the "common-sense" point of view we normally use to understand the everyday world around us. In the mid-thirties this dissonance was pinpointed and sharpened up by two famous papers, one by Erwin Schroedinger introducing his notorious cat and the other by Einstein, Podolsky and Rosen ("EPR") on the description of two particles which originate from a common source but are subsequently widely separated in space; however, since neither paper suggested any actual experiment which might discriminate between the two conceptual schemes, most physicists continued to regard this kind of issue as "(merely) philosophical".

This situation changed radically in 1964, when the late John Bell formulated a famous theorem which demonstrates that not only are the two schemes conceptually inconsistent, but so are the experimental predictions they make. More precisely, Bell (and following him Clauser et al., usually referred to as CHSH) considered a whole class of "common-sense" theories about the world which are characterized by two basic postulates, namely:
(1) "realism": isolated physical systems, including single particles, possess definite properties irrespective of whether or not these properties are measured.
(2) "locality": a space-time event A can causally influence a second event B only if A lies in the backward light cone of B.
The class of theories which embody postulates (1)and (2) is usually known as "local realism" (hereafter LR). These authors then considered the experimental setup shown schematically in Fig. (1), (in effect, the setup originally considered by EPR), in which correlated pairs of particles are produced at the source S and fly off back-to-back; each particle is then switched at random into one of two detectors, which measure two different properties, in the case of particle 1 A or A', in that of particle 2 B or B'; it simplifies the argument if we choose these quantities in such a way that in each of the four

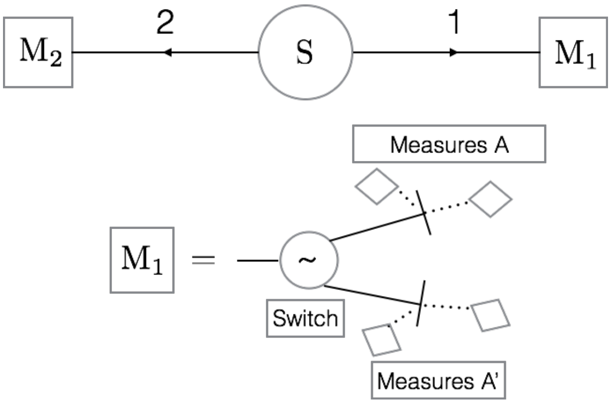

Figure 1: Schematic Set-up of Bell-EPR experiment.

cases the only values found experimentally are +1 or -1. (For example, in the case of two spin 1/2 particles, a possible choice (corresponding to the one made by EPR) would be to take A (B) to be (up to an appropriate constant) the z-component of spin of particle 1 (2), and A' (B') to be the x-component; note that in QM the operator corresponding to A fails to commute with A', and similarly for B and B', but the operators corresponding to A and A' both commute with those representing B and B'. An essential feature of the setup considered is that the events of detection at M_1 and M_2 are spacelike separated from one another and that the choice of what to measure is in each case spacelike separated from the emission event. We then consider the experimentally measurable quantity:

$$K \equiv < AB > + < AB' > + < A'B > - < A'B' > \tag{1}$$

where $< AB >$ means the correlation of the values of A and B as measured on those pairs for which particle 1 was switched into counter A and particle 2 into counter B; more technically,

$$< AB > \equiv \frac{\big(N(++) - N(+-) - N(-+) + N(--)\big)}{N_{totAB}} \tag{2}$$

where N(++) is the number of the "AB-switched" pairs on which both A and B were measured to be +1 (and similarly for $< AB' >$, etc.) and N_{totAB} is the total number of "AB-switched" pairs. What CHSH, following Bell, were able to show is that for arbitrary choices of the quantities A,B,A',B' the quantity K as calculated under the assumption of LR satisfies the inequality

$$|K|_{LR} \leqslant 2. \tag{3}$$

The proof is sufficiently simple to be given here: For any particular pair, by postulate (1), the quantities A,B,A',B' exist and take the values ±1; moreover, by postulate (2), in view of the spacelike separation of the events at M_1 and M_2 the values of A and A' cannot be sensitive to whether it is B or B' which is measured on particle 2, and vice versa; hence the quantity A has the same value when it occurs in the combinations AB and AB' (etc.). It is then a matter of grade-school arithmetic* to show that for any given pair we have the inequality

$$-2 \leqslant \; AB + AB' + A'B - A'B' \; \leqslant 2 \tag{4}$$

Moreover, although the ensembles over which we need to average in order to derive from (4) the experimentally testable inequality (3) are strictly speaking different for the various terms in (1), the difference resides only in events (measurement choices and their consequences) which are spacelike separated from the emission at S (and from one another), so that in view of postulate 2 we may treat these ensembles as identical. Thus the average over (4) indeed yields (3), QED.

Had the inequality (3) (which, I emphasize, is a prediction concerning *experimental results*) been noticed in say 1900, I would bet that it would not have been thought worth testing in an actual experiment, since it is so obviously true! The fundamental observation made by Bell is that for certain choices of the measured quantities A,B...(interestingly, not those made by EPR) and certain kinds of initial state, the predictions of quantum mechanics do not satisfy (3). This observation is in some sense the basis of the whole science now known as "quantum information", and "Bell's theorem" has acquired such fame that there is now a street in his native Belfast named after it**.

The first experiment which consciously set out to test the predictions of QM vis-a-vis those of the class of local-realistic theories was that of Freedman and Clauser in 1972, and since then there have been literally hundreds of experiments which have improved on the latter in one way or another. These experiments all use the setup shown schematically in fig.1; the vast majority have employed photons as the "particles" in question. With a handful of exceptions in the early days which are now thought to be understood, these experiments have uniformly given results which are not only consistent with the predictions of QM but inconsistent with those of LR, sometimes by over 100 standard deviations. So at first sight the issue is settled: Nature does not believe in local realism!

Alas, life is not quite that simple. While the conclusion that the experimental data are consistent with QM is not problematical, the inference that LR is excluded is complicated by the fact that in various respects (known in the literature as "loopholes") the real-life experiments do not always conform to the idealized model described above. The three most commonly discussed loopholes are as follows: (1)"Locality": the event of switching particle 1 into counter A or A' may not be spacelike separated from the event of detection (or switching) of particle 2, allowing the possibility of causal communication between them; moreover, the "switching" may not be spacelike separated from the emission. (2) "Freedom of choice": the switching may not be truly "random", leaving open the possibility that (e.g.) the ensemble of pairs on which A and B are measured is not identical to that on which we measure A' and B, thus invalidating the inference from (4) to (3). (3) "Detection": because of the imperfect efficiency of the detectors, the measurement of (say) A does not always yield +1 or -1: the photon may simply not be detected in either of the channels shown in fig.(1),and this cannot be incorporated into the above proof as it stands (but see below).

Rather surprisingly, while many existing experiments have blocked one or two of the loopholes, there is none up to now which has blocked all three simultaneously, so that a sufficiently determined advocate of LR could argue that maybe Nature is playing a sophisticated trick on us. However implausible one may feel this scenario is, it is clearly important to implement a completely "loophole-free" experiment, and this has now been done independently by three different groups; in each case these are multi-institutional, so I shall use the affiliation of the first author to refer to them respectively as the Delft, NIST and IQOQI experiments. In reading these papers (especially the Delft one), the following point needs to be born in mind: In the context of exclusion of LR theories (as distinct from that of verifying QM) it is absolutely irrelevant how the correlations between the properties of particles 1 and 2 are generated; the mere fact that these correlations are found experimentally to violate the inequality (3) (or a similar one, see below) is sufficient to exclude LR.

In all three experiments (as in many earlier ones) the "locality" loophole is closed simply by situating the measurement stations 1 and 2 sufficiently far apart from one another and from the source and making both the "switch" and the measurement sufficiently fast; the distances between M_1 and M_2 were 58 m for IQOQI, 185 m for NIST and 1.3 km for Delft. In all cases the "freedom-of-choice" loophole was blocked by conditioning the choice of measurement (the "switch") on the output of a quantum random number generator. As regards the "detection" loophole, the Delft experiment (in which the "particles" are actually the spins of two nitrogen-vacancy complexes in diamond, which are correlated by being entangled with photons whose state is then postselected) simply had a high enough detection efficiency (96-97%) that they could apply the original CHSH inequality (3). By contrast,the NIST and IQOQI experiments relied on a modified version of (3) due to Eberhard, which requires only that for any given choice of settings each detector records either the arrival of a particle (labelled +) or its non-arrival (labelled 0). (Of course, one must then know for sure that a pair was emitted,and this is guaranteed by the use of auxiliary so-called "event-ready" detectors). The modified inequality reads

$$J = p(++|ab) - p(+0|ab') - p(0+|a'b) - p(++|a'b') \leqslant 0. \tag{5}$$

where (e.g.) $p(+0|ab)$ means the experimentally measured probability that with detectors set to detect values of A and B equal to +1 (only), detector 1 fires and detector 2 does not. The inequality (5) does not require any assumptions about detector efficiencies, and the algebra necessary to prove it in LR theories is only slightly more complicated than that required for 2). The inequalities (3 and (5) refer to statistical averages over notional infinite ensembles, so that the fact that they are violated in an experiment with a finite number of runs (trials) cannot strictly speaking prove LR to be false; all it can establish is that the probability of it being true is very small. In the Delft experiment the average value obtained for the quantity K of Eq. 1 is 2.42±0.02 (actually slightly exceeding the QM prediction 2.30±0.07); since the number of trials was fairly small ($o(250)$) the calculated probability of obtaining this result in LR is 0.019 (or 0.039 if one allows "memory" effects in the detector). The NIST and IQOQI experiments both used photon pairs essentially in the configuration of fig.1, and tested the inequality (5) rather than 3; while in each case the positive value of J obtained was extremely small (o(2×10^{-7}) for NIST, o(7×10^{-7}) for IQOQI), the large number of trials allowed the authors to conclude that the probability of obtaining this result in a LR theory because of a statistical fluke was extremely small (less than 2.3×10^{-3} for NIST, and less than 10^{-30} [sic!] for IQOQI). So it looks as if local realism is dead....

Is that right? Are there any more subtle loopholes out there? One possibility which some of these papers mention is that the output of the quantum random number generator is not really "random" but somehow predetermined, in such a way as to give the apparent violation of MR; it is not clear how one could test such an assumption. A second relates to the assumption, made explicitly or implicitly in each of the papers, that a definite outcome is realized at some specified point in the "measurement" process (despite the fact that QM says it is not - cf. the Schroedinger cat paradox). I believe that in the distant future it may be possible to test this hypothesis by demonstrating the interference (or not) of different "measurement" outcomes. Finally there is a subtle point relating to the definition of "local realism": as defined by postulate (2) above it is actually an amalgam of two postulates, which prohibit respectively superluminal causality and backward-in-time causality; while within the framework of special relativity the former entails the latter, there seems to be no particular reason for this to be so in a more general theory. Thus the experiments cannot exclude a scenario in which, despite its being "local" in the sense of not permitting superluminal transmission of causal effects, the outcomes of the measurements propagate backwards in time and affect either the settings or the conditions at the source. However,such a scenario would be very "weird",and it is not clear to me that it is experimentally testable.Thus I imagine that most physicists will conclude from these three experiments that Nature indeed does not believe in local realism.

*At least if the grade school in question is outside the US.
**Apparently a Belfast by-law forbids the naming of streets for individuals, living or deceased.