Spontaneous oscillations in quantum systems

Floquet Time Crystals
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Recommendation and Commentary by Leon Balents, KITP, UCSB

Spontaneous symmetry breaking is a fundamental paradigm in physics, from the Higgs field of the standard model to its many manifestations in condensed matter and materials physics. It occurs whenever a symmetry of the equations of motion, or the Hamiltonian for Hamiltonian dynamics, is not fully preserved by physical quantities. The broken symmetry can be internal or global, such as spin-rotation symmetry which is broken across the Curie point of a ferromagnet, or a spacial symmetry such as translations which are broken from their continuous form in a fluid or gas to a discrete subgroup in the transition to a crystalline solid.

A seemingly more exotic idea is time-translation symmetry breaking (TTSB). By virtue of the analogy with spatial symmetry breaking, such a situation was deemed a "time crystal" by Wilczek[1]. In this language it appears very exotic, but it can be recast in more familiar form. Any oscillator is in a sense an example of TTSB: a translation by less than the period of the oscillator alters the configuration of the oscillator. Oscillation obviously occurs easily in finite classical and quantum systems – c.f. the simple harmonic oscillator. The trickiness comes if you want the oscillation to be a robust, universal feature of a system.

To wit, most of our understanding of universality rests on equilibrium statistical mechanics. Unlike the more conventional forms of symmetry breaking, persistent oscillations are not present in equilibrium, almost by definition: in equilibrium all observables settle down to average values determined by the rules of statistical mechanics. A recent cogent discussion is in Ref.[2]. Hence, a system with TTSB must be out of equilibrium. This in ensured by *driving* with external forces, influx of energy, etc. Then spontaneous oscillations can certainly arise by various mechanisms. For example in the AC Josephson effect, driving a Josephson junction above its critical current leads to oscillations of voltage. There are well-known oscillating chemical reactions.

Such mechanisms explain spontaneous oscillations at short times, but not their coherence. Naively small perturbations or noise can induce phase shifts that build up over long times, spoiling the perfect phase coherence. The formal question, analogous to that in ordinary spontaneous symmetry breaking, is whether the oscillations remain synchronized over long time and space separations, i.e. is there "long range order"? Again, in classical systems there is a long history of asking this question, from influential work by Winfree on biological rhythms[3] to studies of narrow band noise in charge density waves[4].

In the highlighted paper, Else *et al* address the existence of TTSB in quantum systems, with driving perturbations periodic in time. Since the underlying symmetry of the dynamics in this case is already discrete, TTSB must also be discrete – it manifests if physical quantities oscillate with a period larger than that of the drive. Any finite system of this type has eigenstates of Floquet type: states where $|\psi(t+T)\rangle = e^{i\phi}|\psi(t)\rangle$, where T is the period of the drive. This is the analog of a stationary state in Hamiltonian mechanics, and obviously in such a state expectation values are invariant under translations by T. So there is no TTSB in a Floquet eigenstate.

However, it is not obvious that the dynamics of a generic state behaves the same way, and in fact Else *et al* construct an example where they do not. Specifically, they present a simple model of spins in which the unitary evolution over a period T consists of two parts, $U(T) = U_2 U_1$. They take $U_1 = \prod_i \sigma_i^x$, which flips every spin in the z basis, and $U_2 = \exp[iH(\{\sigma_i^z\})]$, where H is a local Hamiltonian-like function, so that evolution by U_2 assigns a state-dependent phase to each product state in the z basis. It is straightforward to show that consequently all the Floquet eigenstates are Schrödinger cat states, i.e. they are a superposition of two macroscopically distinct components with all flipped spins in one component relative to the other. Because such a cat state is exponentially difficult to construct from a product state, they argue that a generic initial state never relaxes to a Floquet-like state, and instead undergoes what is basically a *persistent Bloch oscillation* living for a time that grows exponentially with system size. The situation has close analogies to the usual symmetry breaking in an Ising ferromagnet, in which the true finite system ground states are cat states, but are never reached in physically relevant times.

This establishes a very simple example of TTSB in a peridiocally driven quantum system, and a nice connection of TTSB to non-local entanglement. The authors further focus on the case in which the phase factor derives from a strongly disordered Hamiltonian, in which the undriven Hamiltonian would exhibit many body localization[5]. In that context, they argue that many body localization lends stability to TTSB. Like most results for many body localization, the stability argument is not rigorous, but it is reasonable, and indeed they present numerical results consistent with this claim. Moreover, the new work shows that prior theoretical studies giving instances of MBL phases which symmetry protected topological order or discrete symmetry breaking also exhibit TTSB (see the highlighted articles Refs.[24-30]).

One may wonder whether this strong disorder regime is the *only* situation where TTSB is stable, or whether there may be other examples. Are there examples of TTSB beyond the simplest discrete multiplication of the drive period? Assuming TTSB indeed exists, then there should be dynamical phase transitions into or out of the oscillating situation, which would also be interesting to study. On a complementary front, the simplicity of discrete TTSB seems a practical target for experiments with ultra-cold atoms or other driven quantum systems. Despite the current obsession of theory with topology, it seems that the old fashioned notion of symmetry breaking still has some surprises left for the community.

References

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