Spinning Spins

1. **Barnett effect in paramagnetic states**
   Authors: Masao Ono, *et al.*
   Phys. Rev. B **92**, 174424 (2015)
2. **Observation of Barnett fields in solids by nuclear magnetic resonance**
   Authors: Hiroyuki Chudo, *et al.*
   Applied Physics Express **7**, 063004 (2014)
3. **Rotational Doppler Effect and Barnett Field in Spinning NMR**
   Authors: Hiroyuki Chudo, *et al.*
   J. Phys. Soc. Japan **84**, 043601 (2015)
4. **Spin hydrodynamic generation**
   Authors: R. Takahashi, *et al.*
   Nature Physics **12** 52 (2016)

Recommended with a Commentary by S. M. Girvin, Yale University

Several interesting recent papers connect spin angular momentum and mechanical motion. It is a familiar fact that a spin in a magnetic field having a Hamiltonian $H = \gamma\vec{B}\cdot\vec{S} = \gamma BS^z$,behaves like a gyroscope and, due to the torque trying to align the spin magnetic moment with the field, precesses about the axis of the magnetic field (in this case, the z axis) with angular frequency $\Omega = \gamma B$where γ is the gyromagnetic ratio. It is also a familiar fact if we jump into a new frame rotating at rate ω relative to the (inertial) laboratory frame, via the time-dependent unitary transformation

$$U(t) = e^{-\frac{i}{\hbar}\omega S^z t}, \tag{1}$$

the apparent precession frequency changes to $\Omega' = \Omega - \omega$.This is sometimes referred to as the rotational Doppler effect. In magnetic resonance experiments, the analysis is often simplified by going to the co-rotating frame by choosing $\omega = \Omega$ in which the precession comes to a halt, as if the external magnetic field had gone to zero. Flipping the spins is done via a resonant transverse RF field which in the laboratory frame is oriented in the x direction (say)

$$\vec{b}(t) = b_0(t)(\cos\omega t, 0, 0) = \frac{b_0}{2}\left\{(\cos\omega t, \sin\omega t, 0) + (\cos\omega t, -\sin\omega t, 0)\right\} \tag{2}$$

where $b_0(t)$ is the slowly varying envelope of the RF magnetic field. The second equality resolves the linear oscillations in the x direction into two components rotating with opposite angular velocities. Jumping into the rotating frame Doppler shifts one of these to zero frequency and the other to 2ω. Since it is far off-resonance (i.e. which is zero frequency in the rotating frame), the latter 'counter-rotating' field is often neglected in the so-called rotating wave approximation. The Hamiltonian in the rotating frame then becomes

$$H_R = \gamma \frac{b_0(t)}{2} S^x. \tag{3}$$

The resulting precession about the (rotating frame) x axis causes the z component of the spin to undergo Rabi oscillations (in both the rotating and lab frames).

External RF drives are not the only mechanism for flipping spins. Spin-orbit coupling can also transfer angular momentum between the spins and the lattice. This is the mechanism behind the Einstein-deHaas effect in which a freely suspended magnetic material begins to mechanically rotate when it is cooled below the Curie temperature. The sign of the spontaneous magnetic order determines the sign of the mechanical rotation needed to conserve angular momentum. Energy conservation is a separate matter, but in a metal the kinetic energy of the fermi gas of electrons is available as a reservoir.

Spin-orbit coupling arises from a relativistic analysis of the Dirac equation. The analysis is often presented in the following simple way. An electron moving through a solid experiences an electric field due to Coulomb interactions with the nuclei. Jumping into boosted frame co-moving with the electron transforms the E field partly into a B field which couples to the magnetic moment of the particle. A more general analysis [1] of the Dirac equation for a free particle viewed from a non-inertial accelerating frame shows that the acceleration couples to spin.

Similarly, the rotation of a non-inertial frame also produces coupling to the spin. This leads to the Barnett effect which is the dual of the rotational Doppler shift–physically rotating a sample relative to an inertial frame produces a pseudo-magnetic field which attempts to align the spins. By rotating a paramagnetic sample at a tremendous rate (~ 1 kHz!) Ono *et al.* observed spontaneous magnetization of the sample as measured by a nearby magnetometer stationary in the lab frame. One might ask how the spins know they are in a non-inertial frame. Presumably the answer is again the spin-orbit coupling which attempts to equilibrate the spins with the mechanically rotating lattice. In a different setup Chudo, *et al.* observed the Barnett effect in solid samples rotating at ~ 10 kHz via the shift in nuclear magnetic resonance frequency. Interestingly in this the experiment the sample could be held stationary while the detection coil was spun or the sample could be co-rotating with the detection coil. This setup allowed the authors to separate the Barnett effect from the rotational Doppler shift.

In another class of experiments by Takahashi, *et al.* voltages were induced in a liquid metal by flowing it through a narrow channel. The resulting mechanical velocity field has a curl (i.e. rotation) which induces spin currents. The spin currents induce electrical currents via spin-orbit coupling (the inverse spin Hall effect) which in turn induce measurable voltages. This hydrodynamic electrical generation represents a novel form of 'mechanical fluid spintronics.'

References

[1] 'Inertial effects of a Dirac particle,' Friedrich W. Hehl and Wei-Tou Ni, Phys. Rev. D **42**, 2045 (1990).