

Time crystals without localization?

Observation of discrete time-crystalline order in a disordered dipolar many-body system

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Crystalline order is associated with the spontaneous breaking of the continuous translation symmetry of space down to a discrete subgroup i.e. the *Hamiltonian* of the system is invariant under continuous spatial translations, but the *state* is only invariant under a discrete set of translations. Wilczek proposed an extension of this concept [1] to states that spontaneously broke *time* translation invariance - a phase that was dubbed the ‘time crystal.’ Subsequent work established that the ‘time crystal’ could not arise as a stable phase in thermodynamic equilibrium [2]. Interest in ‘time crystals’ has however been renewed through insights from the study of *non-equilibrium* quantum matter. In particular, there is now a large theory literature (see [3] for a review) on time crystals in periodically driven ‘Floquet’ systems, where *discrete* time translation symmetry is spontaneously broken to a smaller subgroup.

Two recent experiments [5, 4] appear to have observed this phenomenology in the *laboratory*, in disordered Floquet systems, which is the one place that theory suggests such phenomenology could arise [3]. Of these two experiments, while [5] works with a system of ten trapped ions, and appears to be well described by the existing theory, the experiment [4] throws up a conceptual puzzle, which is the focus of this review.

The experiment [4] works with an ensemble of nitrogen-vacancy (NV) centers in diamond. In the presence of external magnetic field each NV center can be modelled as a two level system (i.e. a pseudospin), and the ensemble is described by an effective Hamiltonian

$$H = \sum_i \Omega_x(t) S_i^x + \Omega_y(t) S_i^y + \Delta_i S_i^z + \sum_{ij} \frac{J_{ij}}{r_{ij}^3} (S_i^x S_j^x + S_i^y S_j^y - S_i^z S_j^z) \quad (1)$$

where Δ_i is a random onsite field, r_{ij} is the distance between two NV centers, J_{ij} is an orientation dependent co-efficient for the dipolar interaction, and $\Omega_{x,y}$ can be controlled by the microwave field. The particular experimental protocol in question starts with a state polarized in the $+x$ direction, applies a microwave field along the \hat{x} direction $\Omega = \Omega_x$ for a time τ_1 , and then a field along the y direction $\Omega = \Omega_y$ for a time $\tau_2 = \theta/\Omega_y$. This procedure is repeated n times (yielding a periodic drive with period $T = \tau_1 + \tau_2$). Finally, the polarization along the x axis is measured at time nT .

It is observed that for short τ_1 , and for near perfect $\tau_2 \approx \pi\Omega_y$, the polarization $P(nT)$ alternates between positive and negative values, exhibiting $2T$ periodicity i.e. period doubling occurs. This effect is of trivial origin - each period implements a perfect π pulse (flipping the x component of the spins). However, this period doubling occurs because of fine tuning of τ_2 , and detuning τ_2 slightly away from $\pi\Omega_y$ causes the effect to disappear. Remarkably, however, increasing τ_1 by an order of magnitude (which one would think allows the spins to precess more) makes the period doubling robust against detuning of τ_2 , suggesting emergence (at large τ_1) of a robust time crystal phase. A ‘crystalline fraction’ can also be defined by constructing the Fourier transform of $P(nT)$ and examining the spectral weight at $\nu = 1/(2T)$. A refinement of the experimental protocol making use of all three spin states of the NV center is also able to realize an apparently stable phase exhibiting period *tripling* i.e. a Z_3 time crystal. However, in my view the experiment is remarkable for the *open* problems that it shines a light on.

Firstly, while period multiplication is strongly suggestive of time crystalline order, it is *not* conclusive - in particular it does not crisply distinguish a time crystalline phase from trivial examples such as e.g. a set of uncoupled spins which are rotated every period by $2\pi/n$ about some fixed axis. Sharper diagnostics of time crystallinity do exist in the theory literature - I mention in particular [6], which introduces an *order parameter* for time crystallinity in terms of connected correlation functions of operators that are far separated in a thermodynamically large system i.e.

$$\lim_{|\vec{r}-\vec{r}'|\rightarrow\infty} \lim_{L\rightarrow\infty} [\langle O(\vec{r}, nT)O(\vec{r}', 0) \rangle - \langle O(\vec{r}, nT) \rangle \langle O(\vec{r}', 0) \rangle] \quad (2)$$

where $O(\vec{r}, nT)$ is an operator in the Heisenberg representation at position \vec{r} and time nT , and L is the system size. In a time crystalline phase, this *spatiotemporal* correlator should be non-zero for operators O that transform non-trivially under time translation by T , but trivially under some subgroup (e.g. translation by nT with $n > 1$). This definition *does* exclude trivial examples, and includes a thermodynamic limit (as must any definition of a true phase of matter). However, detection of this spatiotemporal order remains an open challenge for experiments.

In addition, the experiment throws up a major *theoretical* puzzle. The theory of time crystals ([7, 8]) relies on *many body localization* (MBL) to protect the order [9]. However, general arguments [10, 11, 12] suggest that a Hamiltonian with $1/r^3$ terms is ‘too long ranged’ to admit of a locator expansion, and thus (in the conventional understanding) cannot be localized. The basic argument is due to Anderson, and works as follows [10] - consider a Hamiltonian with a $1/r^\alpha$ hopping term. In a ball of radius R there are R^d states to hop to, with a typical energy splitting $\sim R^{-d}$, whereas the typical matrix element is $1/R^\alpha$. Hopping occurs if the matrix element exceeds the level splitting. If $\alpha < d$ then one can always find another state to hop to (for large enough R), and so localization is impossible. The case $\alpha = d$ is critical according to this argument, but is still expected to delocalize [11]. A refinement of this argument by Burin [12] states that for *two body* ‘interaction’ terms that fall off as $1/r^\alpha$, it is $\alpha < 2d$ that leads to delocalization, because the number of available two body rearrangements in a ball of radius R grows as R^{2d} . Both by the Anderson and the Burin criteria, dipolar interactions in three dimensions should be ‘too long ranged’ to admit of a localized phase. However, in the theoretical understanding, localization is necessary to

stabilize the time crystal phase. How then is the experiment observing robust time crystalline order?

A number of possibilities suggest themselves. The most prosaic explanation is that the ‘long range resonances’ that destroy localization [10, 11, 12] simply are not relevant on experimental timescales, but would indeed destroy localization (and time crystallinity) on longer timescales. Alternatively, it is possible that the time crystal phase is inheriting its stability through physics independent of many body localization - but that would require the development of a new theoretical picture of time crystalline order. Most exciting to my mind is the possibility that the obstruction to the ‘locator expansion’ posed by long range Hamiltonians reflects simply a breakdown of our perturbative theoretical *tools*, and that the phenomenon of many body localization itself is more robust than the tools that have been developed to describe it, such that the experiment can be *accurately* described using the ‘many body localized’ theoretical formalism. This would then beg the question of what are the more robust tools needed to describe localization in this setting? A resolution of which of these explanations obtains will require careful developments of the experiment and the related theory. Regardless of the particular explanation, the experiment has shone a light on the continuing deficiencies in our understanding of disordered systems with *power law* long range interactions.

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