Angels & Demons: Majorana & Dirac fermions in a quantum Hall edge channel

1. **A mechanism of $e^2/2h$ conductance plateau without 1D chiral Majorana fermions**
   Authors: Wenjie Ji and Xiao-Gang Wen
   arXiv:1708.06214

2. **Disorder-induced half-integer quantized conductance plateau in quantum anomalous Hall insulator–superconductor structures**
   Authors: Yingyi Huang, F. Setiawan, and Jay D. Sau
   arXiv:1708.06752

Recommended with a Commentary by Carlo Beenakker, Instituut-Lorentz, Leiden University

The discovery of *Chiral Majorana fermion modes in a quantum anomalous Hall insulator–superconductor structure* (arXiv:1606.05712) has been widely discussed since its publication in Science,* including a commentary by Jason Alicea in the July issue of the Journal Club. As it should be, a breakthrough announcement such as this is being scrutinized for alternative interpretations. In the case of the 2012 report of a Majorana zero-mode such critical examinations have helped a lot in strengthening the experimental claim, and I presume the same will happen here. It is in that constructive spirit that I wish to discuss the two preprints by Ji & Wen and by Huang, Setiawan & Sau, which address the question: *How unique a signature of Majorana fermions is a conductance plateau at half the conductance quantum?*

To appreciate the issue, we recall that in this context a quasiparticle is called a Majorana fermion if it is a coherent equal-weight superposition of a Dirac fermion and its antiparticle (electron e and hole h). The Majorana fermion is charge-neutral, but so is an incoherent equal-weight mixture of electrons and holes. Since the former has been dubbed the *"angel particle"*, one might stay true to this theme and call the latter a *"demon"*. As explained in Figure 1, a conductance measurement cannot distinguish between the current carried by the coherent superposition $\psi_{\text{angel}} = 2^{-1/2}(|e\rangle + |h\rangle)$ and the incoherent mixture $\rho_{\text{demon}} = \frac{1}{2}(|e\rangle\langle e| + |h\rangle\langle h|)$ — both give rise to a conductance of $\frac{1}{2}e^2/h$.

For a Majorana fermion the equal weight property is fundamental: it is required by electron-hole symmetry at the Fermi level. The incoherent electron-hole mixture is not so constrained, and one might wonder how reasonable it is to assume a charge-neutral mixture.

*Qing Lin He *et al.* Science **357**, 294 (2017).

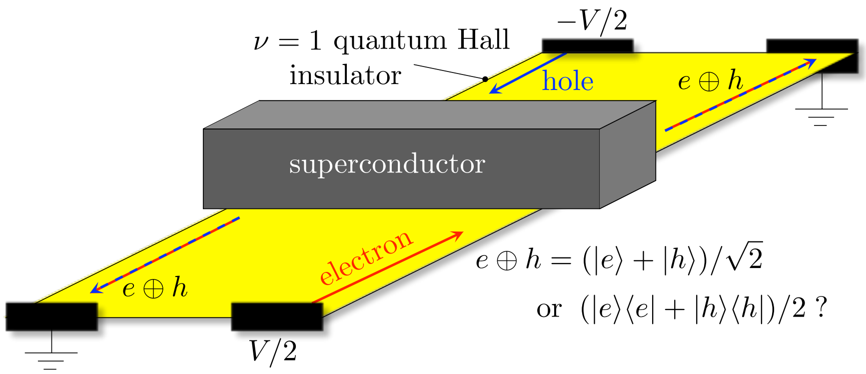

Figure 1: Illustration of spin-polarized ($\nu = 1$) quantum Hall edge channels transmitted and reflected by a superconductor. Application of a voltage $\pm V/2$ between the two ends of the Hall bar injects a current $\pm G_0 \times V/2$ of electrons and holes into the counterpropagating (chiral) edge channels, which are then drained to ground in a superposition. If the superposition has equal electron-hole weight, it carries no current and the total current through the Hall bar is $I = G_0 \times V/2$, resulting in a conductance $G = I/V = G_0/2$ of one-half the conductance quantum $G_0 = e^2/h$ — irrespective of whether the superposition is coherent or incoherent.

The two preprints consider two alternative mechanisms, which in final analysis amount to the same equilibration effect.

Huang, Setiawan, and Sau consider the randomizing effect of multiple Andreev reflections at the superconductor. The charge of a Dirac fermion is $q = e\cos\theta$, where $e^{i\theta}$ performs a random walk on the unit circle as the quasiparticle propagates through the area of length L covered by the bulk superconductor. The average charge becomes exponentially small with increasing L, but the sample-specific fluctuations are large: $\langle (q/e)^2 \rangle = 1/2$, so the $e^2/2h$ plateau would be fully obscured by these “Universal Conductance Fluctuations” — unless there is a self-averaging mechanism. Thermal averaging is rather ineffective (it would only reduce the UCF algebraically by $1/L$), more effectively one could invoke dephasing and equilibration by coupling of the chiral edge mode to the bulk superconductor. I note that the absence of UCF on a fractionally quantized plateau in a graphene p-n junction was explained in this way (arXiv:0704.3608).

More generally, as discussed by Ji and Wen, the self-averaging needed to obtain an effectively charge-neutral current could be provided by local equilibrium of the edge modes with the bulk superconductor. This would generically produce an $e^2/2h$ conductance plateau because the local equilibrium would ensure that the total resistance is the series resistance of two h/e^2 quantum Hall resistors. In the experimental paper this series-resistance scenario was examined and dismissed because the $e^2/2h$ conductance plateau exists only over a narrow magnetic field range of about 10 mT — in a larger field range up to 1 T the conductance equals e^2/h. Both theoretical preprints argue that a percolation transition in the quantum Hall insulator could account for the narrow field range.

In closing I would like to suggest an experiment that can distinguish between a coherent and an incoherent electron-hole superposition— with only a minor modification of the experimental setup and measurement technique. Figure 2 shows an implementation of the

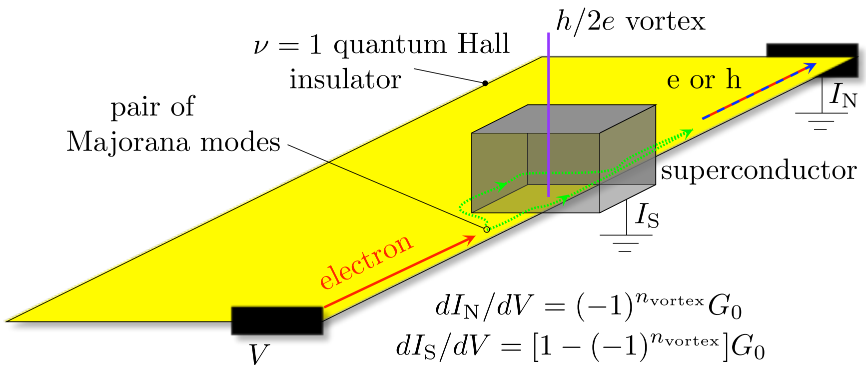

Figure 2: Proposed modification of the experimental design of the previous figure, where the superconductor only covers one edge of the quantum Hall insulator and is grounded. An incoming electron is split into a pair of Majorana modes and then recombines into either an electron or a hole, depending on whether the two paths enclose an even or an odd number of vortices.

$\mathbb{Z}_2$ interferometer of arXiv:0903.2427 and arXiv:0903.2196. At the Fermi level the electron-to-hole conversion via a pair of Majorana modes is fully deterministic, irrespective of any scattering phase shifts, depending only on the parity of the number of vortices enclosed by two Majorana modes. This signal cannot be faked by any incoherent electron-hole mixture, allowing one to distinguish between "angels and demons".

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