A jump in our understanding of quantum criticality in metals

Exact critical exponents for the antiferromagnetic quantum critical metal in two dimensions
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Recommended with a Commentary by Andrey V Chubukov, University of Minnesota

The behavior of itinerant fermions at $T = 0$ at the verge of an instability into a state with either spin or charge order is a fascinating subject which continue to attract a lot of attention from theorists with background in both condensed matter and high energy. At a quantum-critical point (QCP), a scattering by massless bosonic fluctuations of the order parameter destroys fermionic coherence in dimensions $D \leq 3$, at least in some parts of the Fermi surface (FS), leading to a non-Fermi liquid (NFL) behavior. The description of such a state cannot be reached within a conventional perturbation theory in fermion-fermion interaction and requites non-perturbative methods.

The paper by Schlief, Lunts, and Lee (SLL) presents qualitatively new understanding of antiferromagnetic quantum criticality within the spin-fermion model [1]. The model describes itinerant fermions with the Fermi surface like in high-Tc cuprates (Fig. 1), near a $T = 0$ quantum transition into a metallic antiferromagnetic state with commensurate momentum $\mathbf{Q} = (\pi, \pi)$. It assumes that near a QCP, the dominant interaction between low-energy fermions is the exchange of massless collective bosonic excitations in the spin channel.

The spin-fermion model can be viewed as the low-energy effective theory for interacting fermions, after one integrates out high-energy fermions At its upper cutoff, which is a fraction of the fermionic bandwidth, fermions and their collective spin fluctuations are assumed to behave, respectively, as free quasiparticles, and as propagating paramagnons (bosonic $\chi(q, \Omega_m) \propto 1/(\xi^{-2} + \tilde{\mathbf{q}}^2 + \Omega_m^2/c^2)$, where $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{Q}$ and Ω_m is Matsubara frequency). It has been known from the work by Hertz back in 1976 (Ref. [2]) that a direct perturbation theory in spin-fermion coupling g fails because low-energy fermions give rise to a linear in Ω_m term in the bosonic propagator (the Landau damping). This term overshadows the bare Ω^2 contribution and changes the dynamics of critical bosons.

Several groups, who analysed the spin-fermion model before SLL, argued [1, 3] that the right way to proceed is to solve self-consistently for the Landau damping and the fermionic self-energy. The self-consistent equations can be solved exactly if one formally extends the model to N fermionic flavors and takes the limit $N \to \infty$. The solution yields, at the QCP,

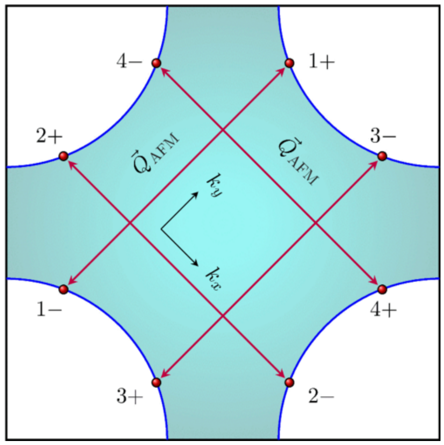

Figure 1: A Fermi surface with fourfold rotational symmetry. The (red) dots represent the hot spots connected by the AFM wave vector.

the bosonic propagator with the dynamical exponent $z_b = 2$: $\chi(q, \Omega_m) \propto 1/(\tilde{\mathbf{q}}^2 + \gamma|\Omega_m|)$, where $\gamma \propto g^2$, and fermionic self-energy $\Sigma(\omega_m, k_\parallel)$, which depends on Matsubara frequency ω_m, and the deviation from a hot spot along the Fermi surface, $k_\parallel$, but not on the momentum component transverse to the Fermi surface. At a hot spot k_h (a FS point for which $k_h + Q$ is also on a FS), the self-energy has a NFL form $\Sigma(\omega_m, 0) \propto \Omega_m^{1/2}$ (Ref. [4]). For other $\mathbf{k}_F$, FL behavior survives, but the quasiparticle residue and the inverse effective mass scale as $k_\parallel$. This in turn yields the specific heat $C(T) \sim T \log T$.

This was initially viewed as a legitimate theory at a QCP (modulo the remark at the end of this commentary). However, the attempts to extend the theory from $N = \infty$ to a finite N (and eventually to the physical case of $N = 1$) ran into problems: already at the leading order in $1/N$ the corrections turn out to depend logarithmically on the energy at which the system is probed [1, 3]. The logarithms appear in three places: in the renormalization of the spin-fermion coupling g, which grows as the system moves to a lower energy, in the renormalization of the Fermi velocity v_{k_F}, which evolves such that the FS gets progressively more nested at a hot spot ($\delta = |(\mathbf{v}_{k_F} + \mathbf{v}_{k_F+Q})/(\mathbf{v}_{k_F} - \mathbf{v}_{k_F+Q})|$ decreases), and in the renormalization of the $\tilde{\mathbf{q}}^2$ term in the bosonic propagator. The last effect is the most crucial one because, as Metlitski and Sachdev explicitly demonstrated [3], it gives rise to the logarithmical flow of the dynamical exponent z_b towards a smaller value.

SLL used these earlier results as an input. They conjectured that at the lowest energies δ becomes small and the corrections to the $\tilde{\mathbf{q}}^2$ term exceed the bare term. Because both the Landau damping term *and* the corrections to $\tilde{\mathbf{q}}^2$ term come from low-energy fermions, they argued that the *full* bosonic susceptibility has to be obtained self-consistently within the low-energy model. They argued that at small δ, the self-consistent equation for $\chi(q, \Omega_m)$ can be obtained by restricting to the disgrams with just one internal bosonic line. The outcome of self-consistent analysis is that the momentum dependence of the bosonic propagator becomes $|\tilde{\mathbf{q}}_x| + |\tilde{\mathbf{q}}_y|$ (modulo logarithms) instead of $\tilde{\mathbf{q}}^2$, i.e., at the lowest Ω_m and $\tilde{\mathbf{q}}$, the dynamical exponent becomes $z_b = 1$. This is a very substantial deviation from the original $z_b = 2$. As the consequence of the change of z_b, the fermionic self-energy becomes less singular at a QCP, and even at a hot spot it has a form similar (but not equivalent) to that in a marginal

FL (Ref. [5]). SLL analysed the structure of corrections to their one-loop self-consistent theory and argued that they remain finite down to zero energy, i.e., $z_b = 1$ theory is stable with respect to perturbations.

In my view, this result is a truly remarkable achievement in the theory of quantum-critical phenomenon. Still, a fundamental question, which is not yet resolved, is whether SLL $z_b = 1$ theory is the end point of the flow that has been detected within $z_b = 2$ theory. SLL argued that it is and have shown in a subsequent paper [6] that they can follow the crossover by working near $D = 3$ instead of $D = 2$. However, (i) the logarithmical flow of z_b from $z_b = 2$ to $z_b < 2$ is specific to $D = 2$ and is not present at $D > 2$ if one keeps ordinary Landau damping $|\Omega|$ term in bosonic propagator (one can get the flow if one modifies the power of Ω), and (ii) at the beginning of the flow the spin-fermion coupling g increases, while in SLL $z_b = 1$ theory it is small, of order δ, How (and if) the two limiting forms of g are connected needs to be understood.

There are also two practical issues. First, if one starts with the theory with a bare δ of order one, logarithmical corrections to $z_b = 2$ scaling develop rather slowly [7], i.e. the $z_b = 1$ behavior emerges at truly low energies. So far, quantum-Monte-Carlo (QMC) studies of the spin-fermion model only found $z_b = 2$ behavior at all values of δ which they analyzed [8] (the QMC analysis has been done for an anisotropic model with only XY spin components, which in principle is different from $SU(2)$ symmetric case). From this perspective, the best way to detect the $z_b = 1$ behavior predicted by SLL is to start with small bare δ. However, if δ is too small, there is another potential problem: a near-nesting of hot fermions gives rise to 1D-type physics [9], which can potentially mask $z_b = 1$ behavior, particularly in QMC studies on a finite set of Matsubara points. Another potential issue is spin-fluctuation-induced d-wave superconductivity. In $z_b = 2$ theory with bare $\delta = O(1)$, superconductivity emerges at an energy/temperature of order of spin-fermion coupling g (Ref.[10]). This scale is generally larger than the one at which $z_b = 1$ behavior emerges. The situation again gets better when a bare δ is small. SLL argued that in this limit superconductivity emerges at an energy, which is parametrically smaller than the upper edge of $z_b = 1$ behavior. Hopefully, advanced QMC and other numerical studies will resolve all these issues, and detect $z_b = 1$ behavior, predicted by SLL.

One last comment. The work by SLL is devoted to an antiferromagnetic QCP, when the order parameter carries the momentum $\mathbf{Q} = (\pi, \pi)$. There is another set of QCP's, for which the order emerges with momentum $Q = 0$, due to Pomeranchuk-type instability. Examples of $Q = 0$ QCP include ferromagnetism, a nematic order that breaks lattice rotational symmetry, and several theories of fermions minimally coupled to $U(1)$ gauge field, like Halperin-Lee-Read composite fermion state at a half-filled Landau level [11]. For these systems, $N = \infty$ theory at a QCP describes fermions with non-Fermi liquid self-energy $\Sigma(\omega) \propto \omega^{2/3}$ and bosons with $z_b = 3$ ($\chi(q, \Omega) \propto 1/(q^2 - i\gamma\Omega/q)$) (Ref.[12]). For this QCP, S-S Lee found [13] that, at three loop level, forward scattering gives rise to $O(1)$ corrections to Σ even at $N = \infty$, and subsequent works have found [14] singular corrections to the fermionic dispersion and the bosonic propagator. For an antiferromagnetic QCP, the corrections of this type do emerge once one considers a composite scattering by two (π, π) spin fluctuations. How this composite scattering affects the crossover from $z_b = 2$ to $z_b = 1$ is another issue for future study.

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