Membrane theory, revisited

Anomalous elasticity, fluctuations and disorder in elastic membranes
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This scholarly treatise is a follow up to pervious articles published in more than twenty years ago[1, 2]. It gives an exhaustive analysis of the theory of membranes. This work is rather timely, because it clarifies a number of issues in the general theory of membranes, and also because of the recent and intensive interest in atomically thin crystals, such as freely suspended graphene, the ultimate crystalline membrane.

The extension of the theory of elasticity to two dimensional systems led to a number of challenging problems. As is well known, when out of plane deformations are neglected, thermal fluctuations prevent truly long range crystalline order at large distances. Moreover, dislocations unbind at a finite temperature, leading to a new kinds of phase transitions, the Berezinskii-Kosterlitz-Thouless transition[3, 4], and also to an intermediate hexatic phase[5].

The situation becomes qualitatively richer when the two dimensional system fluctuates in the out of plane direction. The out of plane motion leads to in plane stretching, and the membrane becomes anharmonic. A simple expansion of the free energy in terms of the in plane displacements, $\vec{\mathbf{u}}(\vec{\mathbf{r}})$, and the out of plane deformation, $h(\vec{\mathbf{r}})$ leads to

$$E = \frac{\kappa}{2} \int d^2\vec{\mathbf{r}} \sum_i \left(\frac{\partial^2 h}{\partial r_i^2} \right)^2 + \frac{\lambda}{2} \int d^2\vec{\mathbf{r}} \left(\sum_i u_{i,i} \right)^2 + \mu \int d^2\vec{\mathbf{r}} \sum_{i,j} u_{i,j}^2 \tag{1}$$

where κ is the bending rigidity, λ and μ are Lamé coefficients, and the strain tensor is

$$u_{i,j} = \frac{1}{2} \left(\frac{\partial u_i}{\partial r_j} + \frac{\partial u_j}{\partial r_i} + \frac{\partial h}{\partial r_i} \frac{\partial h}{\partial r_j} \right) \tag{2}$$

The form of the tensor $u_{i,j}$ is determined by the rotational invariance of the membrane. The terms which include the square of the strain tensor in Eq. (1) give rise to cubic and quartic couplings, which define a non linear, anharmonic theory. A perturbative expansion leads to power law divergences. The scale for which anharmonic effects become relevant is $\ell_{anh} \propto \sqrt{\kappa / \mathrm{Max}(\lambda, \mu)}$. In a typical fluctuating elastic sheet, the bending rigidity increases as t^3, where t is the thickness of the sheet. For many macroscopic systems, defined by a length scale L, the inequality $L \lesssim \ell_{anh}$, is satisfied. Hence, these divergences are not relevant, and the total energy is determined by the first term in Eq. (1). The situation changes in the atomically thin two dimensional materials. For graphene, it has been estimated that $\ell_{anh} \approx 3 - 5$ Å.

The serious divergences of the field theory defined by Eq.(1) were analyzed in the 80's and 90's of the past century[6] (see also[7]). Different techniques, such as the renormalization group, self consistent resummation of diagrams, and numerical techniques were applied. The effect of thermal fluctuations and of fluctuations induced by quenched disorder were analyzed.

The main result of these studies is that the parameters in Eq. (1) were renormalized, and acquire a dependence on temperature, sample size, external tension, and other parameters. in particular, the bending rigidity κ diverges in the thermodynamic limit (infinite sample at finite temperature) while the in plane elastic moduli, λ and μ decrease towards zero, leading to an interesting example of a critical phase in condensed matter physics. This result also implies that a large suspended graphene flake at finite temperature should not be as stiff as generally believed.

Ref.[1] presented a very complete study of the role of thermal fluctuations in a two dimensional membrane, and it characterized in detail the fixed point reached in the thermodynamic limit. It uses a self consistent procedure, equivalent to a resummation of bubble diagrams, similar to $1/N$ methods used in statistical mechanics. The main result is the calculation of the exponents which characterize the parameters in Eq. (1) near the fixed point, $\kappa \sim \kappa_0 (q/q_0)^{-\eta}$, and $\lambda, \mu \propto \lambda_0 (q/q_0)^{2-2\eta}, \mu_0 (q/q_0)^{2-2\eta}$, with κ_0, λ_0 and μ_0 defined at some microscopic scale, and $q_0 \sim \ell_{anh}^{-1}$. The value of η computed in[1] is $\eta \approx 0.82$, which compares rather well with numerical calculations performed later. As analyzed in[8] this good agreement arises from the fact that the approximation in[1] becomes exact in a number of different limits.

The paper discussed here[8] extends significantly the results in[1]. Emphasis is put on the role of quenched disorder. The self consistent resummation described in[1] is generalized to disordered systems, by using the replica method. A new critical exponent, $\eta' \approx 0.45$ is determined for the macroscopic fixed point of a membrane at zero temperature in the presence of quenched disorder. It is found that the temperature is a marginally relevant parameter, so that the fixed point identified in[1] describes the long wavelength behavior at finite temperatures. A wealth of new results is reported for models where the quenched disorder shows long range correlations. This situation is relevant if, for instance, a random distribution of disclinations is present. Depending on the decay of the various types of disorder possible a rich phase diagram emerges. There are phases which are flat on the average, although they contain random wrinkles, as well as completely crumpled phases. There are phases where temperature and disorder effects are defined by the same exponents, so that both play a role, and phases dominated by disorder, where static roughness is frozen. These phases show glassy behavior.

Graphene attracted a significant attention, among other things, because of its very large stiffness[9]. Other angstrom-thin two dimensional materials, such as MoS_2, also show very large elastic moduli[10, 11]. The experimental[12, 13, 14, 15, 16] and theoretical[17, 18] interest in the membrane aspects of graphene and other atomically thin two dimensional materials is large, and growing rapidly. The results reported in[8] are highly relevant to these studies. The properties of the long wavelength fixed point identified in[1] and[8] imply that the in plane elastic moduli of graphene should decrease towards zero in large, suspended samples.

Finally, it is worth mentioning that interesting work on membranes is being done based on a non perturbative treatment of out of plane wrinkles, induced by compressive stresses[19, 20]. It would be interesting to connect these works to the diagrammatic approach in[8].

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