

Mystery solved of the sawtooth Josephson effect in bismuth

1. Ballistic edge states in bismuth nanowires revealed by SQUID interferometry

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2. Higher-order topology in bismuth

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*Recommended with a Commentary by Carlo Beenakker,
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A challenging theoretical problem from the 1960's was to calculate the supercurrent through a clean Josephson junction of length L long compared to the superconducting coherence length ξ . The calculation is complicated because every other energy level in the junction contributes with opposite sign to the supercurrent, and the resulting alternating series has to be summed with great accuracy to finally arrive at the correct answer: A piecewise *linear* dependence of the supercurrent I on the phase difference ϕ , distinct from the sinusoidal dependence in a short junction. The linearity can be understood by noting that the regime $L > \xi$ calls for a local relation between the current density and the gradient ϕ/L of the phase. Because I must be a 2π -periodic function of ϕ , the linear rise $I \propto \phi/L$ is interrupted by a jump when $\phi = \pm\pi$, resulting in a *sawtooth* current-phase relationship.

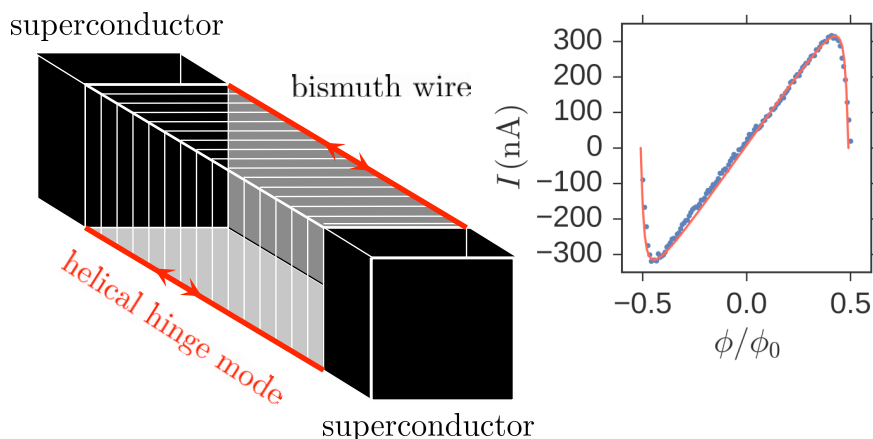
Half a century passed and experimental observation of the sawtooth remained elusive. Disorder suppresses higher harmonics of the ϕ -dependence, leaving only the sinusoidal fundamental when the mean free path ℓ drops below the junction length. The two conditions $L > \xi$ and $L < \ell$ needed for the sawtooth to appear are difficult to combine in a real material.

The report last year ([Murani *et al.*, 2017](#)) by the Paris group of Bouchiat and Guéron of a sawtooth current-phase relationship in a bismuth nanowire was more than a pleasing confirmation of a long-standing theoretical prediction — it was unexpected because the nanowire was strongly disordered ($L = 1.4 \mu\text{m}$, $\ell \approx 100 \text{ nm}$). The authors interpreted their finding as evidence for ballistic conduction along one-dimensional edge states, as in the

quantum spin-Hall effect. Such edge states are known to exist in bismuth bilayers (theory: [Murakami, 2006](#); experiment: [Drozdov *et al.*, 2014](#)), but the 300 nm thick nanowire is far from this two-dimensional regime. While the absence of backscattering over macroscopic distances suggests topological protection, bismuth is not known to be a topological insulator.

A preprint from last month ([Schindler *et al.* 2018](#)), by a collaboration of theorists and experimentalists from ten different groups, solves the mystery: Bismuth is a *second-order* topological insulator.

Recall that a topological insulator combines a gapped 3D bulk with a gapless 2D surface. This applies to bismuth when it is alloyed with antimony, but in a pure bismuth crystal both bulk and surfaces are gapped.* The remarkable discovery by Schindler *et al.* is that different facets of a bismuth crystal can have gaps of opposite sign. When two such facets meet at a hinge the gap must pass through zero, producing a gapless hinge state. The combination of a gapped 3D bulk with gapless 1D hinges is called a topological insulator of second order. (A third-order topological insulator would have gapless 0D corners, see “[Topological Insulators Turn a Corner](#)”.)

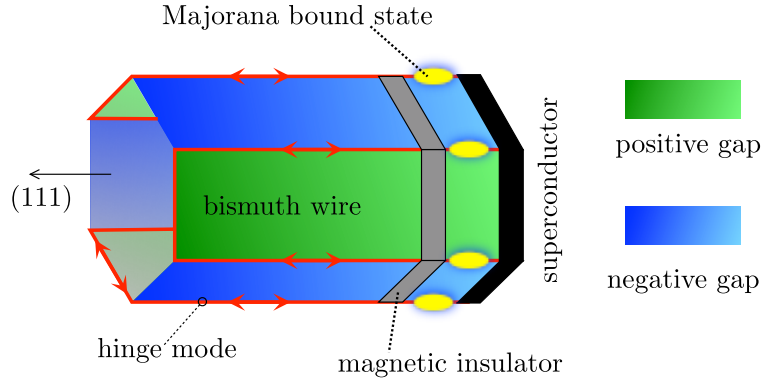


The left panel shows a sketch of a bismuth wire connecting two superconducting electrodes. The gap inversion between the surfaces is indicated by the presence or absence of shading. A gapless mode propagates along each of the two hinges that join surfaces with opposite sign of the gap. The right panel (copied from Schindler *et al.*) shows the sawtooth dependence of the supercurrent I through the wire on the flux bias ϕ , in units of the flux quantum $\phi_0 = h/2e$. (The ratio $2\pi\phi/\phi_0$ is the superconducting phase difference.) The two hinge modes contribute in parallel with slightly different periodicity, so their contributions can be separated by a Fourier transform. The plot shows one contribution.

One can think of this experiment as the 3D generalization of the Majorana-Josephson junction proposed in a 2D quantum spin Hall insulator by [Fu and Kane \(2009\)](#). The predicted 4π -periodicity of the current-phase relationship is not observed (presumably because of quasiparticle poisoning), but there are [more robust signatures](#) of Majorana fermions that one might search for. If the hinge mode is gapped by a magnetic insulator a Majorana bound

*Bismuth is a semimetal rather than an insulator because the Fermi level is not precisely in the gap, leaving small electron and hole pockets. In the Josephson junction experiment this semimetallic conduction is suppressed by disorder.

state is trapped at the interface with the superconductor, which would then manifest itself as a zero-bias conductance peak — similar to the [Majorana signature in InSb nanowires](#), but in a single-mode ballistic system.



As a suggestive fact, we note in closing that a bismuth crystal grown along the (111) direction has 6 facets, so if it traps Majoranas it could naturally realize the “[hexon](#)” proposed as a building block for a topological quantum computer.