A breakthrough in many body quantum chaos

1. **Many-body quantum chaos: Analytic connection to random matrix theory**
   Authors: Pavel Kos, Marko Ljubotina and Tomaz Prosen
   Phys. Rev. X 8, 021062 (2018)
2. **Exact spectral form factor in a minimal model of many body quantum chaos**
   Authors: Bruno Bertini, Pavel Kos and Tomaz Prosen
   arXiv: 1805.00931

Recommended with a Commentary by Rahul Nandkishore, University of Colorado at Boulder

Random matrix theory is the cornerstone of our understanding of quantum chaos. There is by now a mountain of numerical evidence indicating that generic non-integrable quantum systems have energy level statistics governed by random matrix theory * - indeed whether the level statistics obey random matrix theory is one of the most commonly used diagnostics for ergodicity. Nevertheless, analytic derivations of random matrix statistics have until recently only been available for one or few particle systems with a clear semiclassical limit. The ubiquitous applicability of random matrix theory to *many body* quantum systems without a clear semiclassical limit (e.g. spin chains) had until recently defied analytic understanding. This particular fortress has now been breached, by the remarkable highlighted papers that demonstrate how random matrix level statistics may be *derived* in certain clean (translation invariant) many body quantum systems.

The papers in question make two key approximations. Firstly, instead of considering Hamiltonian dynamics (i.e. evolution in continuous time) they consider instead quantum circuits with discrete timestep evolution. Secondly, they specialize to circuits where the gates that are applied are periodically repeated, such that one can apply the tools of *Floquet* theory. That is, the time evolution is generated by a set of gates which repeat periodically every τ timesteps, such that one can consider simply the stroboscopic or 'Floquet' time evolution operator $U(\tau)$, which generates time evolution by one period. It follows from unitarity that the eigenvalues of $U(\tau)$ must be unimodular, and can thus be denoted as $\exp(i\phi_m)$, where the ϕ_m are the *eigenphases* of the problem. The distribution of the eigenphases, in a chaotic system, is expected to follow the *circular* ensembles of random matrix theory.

*Leaving aside special cases like many body localized systems, which are not described by random matrix theory, but which possess a form of emergent integrability

The particular quantity that is examined is the 'spectral form factor' of the Floquet eigenphases. That is, if $\rho(\phi)$ is the normalized density of states of the eigenphases, and $R(\theta) = \langle \rho(\phi + \theta/2)\rho(\phi - \theta/2)\rangle - \langle\rho\rangle^2$ is the connected two point correlator of the densities [†], then the spectral form factor for a system with Hilbert space of dimension $\mathcal{N}$ is given by the Fourier transform

$$K(t) = \frac{\mathcal{N}^2}{2\pi}\int_0^{2\pi} R(\theta)e^{-i\theta t}d\theta \tag{1}$$

Random matrix theory predicts that this spectral form factor should vanish as $K \sim t$ at small t, with a co-efficient that depends on the ensemble in question. The vanishing of $K(t)$ at small t is related to the incompressibility of the spectrum.

Kos, Ljubotina and Prosen are able to calculate the spectral form factor analytically for a particular clean (disorder free) many body quantum circuit, consisting of two timestep evolution. Key to the analysis is that one of the two timesteps is 'non-interacting' (i.e. consists purely of single qubit gates), while the other timestep, while containing abitrarily long range interactions, involves only gates that are diagonal in the computational basis. The authors are then able to map the computation of the spectral form factor to the partition function of an Ising model on a ring of circumference t. In this manner they are able to analytically derive a spectral form factor in agreement with random matrix theory over the window of timescales $t_E < t < t_H$, where t_H is the Heisenberg time of the system (exponential in system size), while the critical time t_E for onset of random matrix statistics is *logarithmic* in system size. A similar logarithmic scaling of the critical time for onset of random matrix statistics was found in [2, 3]. This is a *tour de force* result, corresponding to a analytic derivation of random matrix level statistics in a clean many body quantum system, albeit with long range interactions.

A further breakthrough is presented by Bertini, Kos, and Prosen, who analytically derive the spectral form factor (and show that it agrees with random matrix theory) in a model with interactions that are strictly *local* in real space. The model in question consists of periodically repeated two timestep evolution, where the generating Hamiltonian for the first timestep consists of a nearest neighbor Ising interaction and a longitudinal field, while that for the second timestep consists only of a uniform transverse field. This shares again the feature that one timestep consists only of single qubit gates, while the other consists only of gates diagonal in the computational basis. To define ensemble expectation values it is further convenient to introduce spatial randomness in the longitudinal fields. Once again, Bertini *et al* are able to reinterpret the calculation of the spectral form factor as the transfer matrix evaluation of a partition function, and hence to analytically obtain the spectral form factor (which agrees with random matrix theory predictions) in the interval of times $t_E < t < t_H$. This time, the critical time t_E for onset of random matrix statistics is found to be order *one* in the thermodynamic limit (i.e. lacking the logarithmic divergence observed in the earlier work). As an additional (remarkable) corollary, the results in the thermodynamic limit are found to be independent of the strength of the disorder in the longitudinal fields. This implies that random matrix level statistics may be derived both in the limit of zero disorder (i.e. clean systems), and for arbitrarily strong but finite disorder - constituting an analytic

[†]Here and below, expectation values are taken either over a window of timesteps, or, alternatively over an ensemble of infinitesimally disordered systems

proof of the absence of localization in this model at any finite disorder strength.

Taken together, these works represent a remarkable breakthrough in our understanding of many body quantum chaos. It remains to be seen to what classes of systems the techniques introduced can be applied, and whether the understanding obtained can be generalized to Hamiltonian dynamics (with continuous time evolution) - but the prospect of an *analytic* understanding of the origin of random matrix statistics in many body quantum chaos is in sight. The results also throw up a fascinating conceptual puzzle. Namely, the spectral form factor obtained is that of a *single* many body quantum chaotic system, and is sharply distinct from that of many 'disconnected' quantum systems. However, a thermodynamically large system with purely local interactions develops this spectral form factor in order one time - well before distant parts of the system establish causal contact. How does the system know in so short a time that it is a single many body quantum system?

While I do not discuss them here, the reader is encouraged also to peruse the closely related and contemporaneous works [1, 2, 3].

References

[1] *Solution of a minimal model for many-body quantum chaos* Amos Chan, Andrea De Luca and J.T. Chalker, arXiv: 1712.06836

[2] *Spectral statistics in spatially extended chaotic quantum many body systems* Amos Chan, Andrea De Luca and J.T. Chalker, arXiv: 1803.03841

[3] *Onset of random matrix behavior in scrambling systems* Hrant Gharibyan, Masanori Hanada, Stephen H. Shenker, and Masake Tezuka, arXiv: 1803.08050