Particle-hole symmetry at the half-filled Landau level

1. **Particle-Hole Symmetry in the Fermion-Chern-Simons and Dirac Descriptions of a Half-Filled Landau Level**
   Authors: Chong Wang, Nigel R. Cooper, Bertrand I. Halperin, and Ady Stern
   Phys. Rev. X **7**, 031029 (2017)

2. **Particle-hole symmetry and electromagnetic response of a half-filled Landau level**
   Authors: Michael Levin and Dam Thanh Son
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Recommended with a Commentary by Michael Mulligan, UC Riverside

Motivation

The discovery over 25 years ago of transport anomalies in the lowest Landau level near even denominator filling fractions has revolutionized our understanding for how electrons in the fractional quantum Hall regime can behave (see [1] for a review). In seminal work that generalized the application of statistical transmutation from the arena of the quantum Hall effect [2, 3] to that of compressible states, Halperin, Lee, and Read (HLR) [4] posited that electrons in a half-filled lowest Landau level ($\nu = 1/2$) could be described by composite fermions (CFs) (see also [5]). In this picture, an electron is viewed as a bound state of a CF and two magnetic flux quanta (pointing in a direction opposite to the external magnetic field); since the number of CFs equals the number of electrons, the external magnetic field is completely screened on average by the magnetic flux quanta at $\nu = 1/2$ so that the CFs form a Fermi liquid-like mean-field state. CF mean-field theory has been remarkably successful in describing the qualitative phenomenology of the $\nu = 1/2$ non-Fermi liquid state [1]. For instance, quantum oscillations about half-filling are found to be controlled by the deviation of the external magnetic field from its $\nu = 1/2$ value, instead of the total external field.

Nevertheless, it has remained unclear whether the HLR theory is compatible with Landau level particle-hole symmetry (PH symmetry) that the 2-body electron Hamiltonian enjoys at $\nu = 1/2$ (in the idealized limit where there is no Landau level mixing). Intuitively, PH symmetry says the $\nu = 1/2$ state is equally well accessed by populating the empty vacuum with electrons or draining a filled lowest Landau level with quasiholes. If the HLR theory is to produce a PH symmetric electrical Hall response, $\sigma_{xy} = \frac{1}{2}\frac{e^2}{h}$, the CFs must exhibit a Hall conductivity (with respect to the Chern-Simons gauge field to which they couple),

$$\sigma_{xy}^{\mathrm{CF}} = -\frac{1}{2}\frac{e^2}{h}, \tag{1}$$

when the electrical resistivity $\rho_{xx} \neq 0$ [6]. While there is no symmetry precluding a non-zero CF Hall response in the HLR theory, an $\mathcal{O}(1)$ response (in units of e^2/h) is unexpected of a Fermi liquid-like state in vanishing effective magnetic field. Consequently, Hall transport measurements [7] and numerical experiments [8, 9], which are consistent with an emergent PH symmetry, and refined quantum oscillation experiments [10] away from $\nu = 1/2$ have called into question the precise theoretical description of the state at half-filling.

Dirac Composite Fermions

In a brilliantly insightful paper, Son [11] introduced a new theory for the half-filled Landau level in terms of a Dirac composite fermion. The distinguishing feature of the Dirac CF theory is that it *manifestly* preserves PH symmetry, in contrast to the HLR theory. Its effective Lagrangian:

$$\mathcal{L}_{DCF} = \bar{\psi}(i\partial_\mu + a_\mu)\gamma^\mu\psi + \frac{1}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu A_\rho + \frac{1}{8\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho. \tag{2}$$

Here, ψ is a 2-component Dirac composite fermion; a_μ with $\mu \in \{t, x, y\}$ is an emergent $U(1)$ gauge field; A_μ is a non-dynamical field representing electromagnetism; γ^μ are appropriate gamma matrices. So long as the part of the effective Lagrangian containing only ψ or a_μ preserves time-reversal symmetry, the Hall conductivity is entirely determined by the Chern-Simons term for A_μ in Eq. (2), which contributes the half-integer electrical Hall response. In this way, time-reversal invariance of the dynamical part of $\mathcal{L}_{DCF}$ is identified with PH symmetry of the electron system. (In the interest of full disclosure, it is also necessary to flip the sign of A_t and then add a filled Landau level by shifting $\mathcal{L}_{DCF} \mapsto \mathcal{L}_{DCF} + \frac{1}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$ in order to complete the PH transformation; see [12] for additional discussion.)

PH symmetry prevents the Dirac CFs and the corresponding electron system the Dirac CF theory is meant to describe from becoming a trivial insulator: for instance, a mass term for ψ is odd under time-reversal. Instead, the Dirac CFs must either remain gapless in the IR or realize a state with non-trivial topological order [13, 14]. This conclusion holds even in the presence of disorder, as long as PH symmetry is preserved. Thus, the Dirac CF theory is a manifestly distinct starting point from which to investigate various non-Fermi liquid related behaviors. For instance, the Dirac CF theory has helped to motivate a new candidate ground state at $\nu = 5/2$ that may be relevant to a recent experiment [15].

An IR Equivalence?

Despite the differences in their formulations, there is growing evidence that the HLR and Dirac CF theories have the same long wavelength experimental consequences. To arrive at this conclusion, Wang, Cooper, Halperin, and Stern [16] showed in a beautiful recent paper that it is crucial to consider the behavior of the two theories in the presence of weak

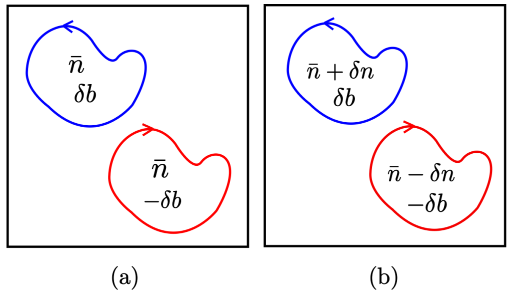

Figure 1: (a) Typical spatial configuration with only magnetic flux disorder; (b) Typical spatial configuration of random CF density and magnetic flux disorder "slaved" to one another as in Eq. (3). In both cases, system has an average CF density equal to $\bar{n}$.

PH symmetric disorder and to include an effect in the HLR theory that had largely been neglected in previous analyses (although see [6]). (PH symmetric disorder means that all odd moments of the chemical potential disorder vanish.) When the electrons see a random chemical potential, flux attachment in the HLR theory implies the CFs experience a perfectly correlated random CF density fluctuation $\delta n(\mathbf{r})$ and effective magnetic field $\delta b(\mathbf{r})$ disorder:

$$-4\pi\delta n(\mathbf{r}) = \delta b(\mathbf{r}). \tag{3}$$

Such "slaved" disorder is *not* generic.

To get an intuitive idea for the effect of Eq. (3) on (disorder-averaged) CF dc transport in the HLR theory, it is helpful to consider the following classical argument [17], the conclusion of which is supported by detailed quantum calculations [16, 17, 18]. Fig. 1 shows cartoons of two distinct types of randomness. Fig. 1 (a) illustrates an incorrect treatment of the HLR theory in which only magnetic flux disorder is present: since for every region with magnetic flux $\delta b > 0$, there is an equal region with the opposite flux $-\delta b$, the Hall conductivity in this system vanishes. By contrast, when random CF density and magnetic flux disorder are both present, the analogous cartoon in Fig. 1 (b) implies the Hall conductivity need not vanish. In fact, when the two disorders are "slaved" to one another,

$$\sigma_{xy}^{\mathrm{CF}} \approx \frac{\nu_{\mathrm{eff}}}{2\pi} = \Big(\frac{\overline{n}+\delta n}{\delta b} + \frac{\overline{n}-\delta n}{-\delta b}\Big) = -\frac{1}{2}\frac{e^2}{h}, \tag{4}$$

where we used Eq. (3) and $\frac{1}{2\pi} = \frac{e^2}{h}$ in the last equality. Various other observables in the HLR theory have also been shown to be consistent with an emergent PH symmetry [16, 19].

While these results hint at a possible IR equivalence of the two CF theories, Levin and Son [20] have derived a remarkable linear relation between the Hall conductivity and susceptibility (with respect to the external scalar potential) at sufficiently low frequencies and wave vectors that any PH symmetric theory must satisfy. To date, it is unknown if the HLR theory is compliant. There are a number of related questions. (i) Why does CF mean-field theory "work" so well? Duality (of which both CF theories are examples) implies this question might be turned around to better understand the nature of the strong electron correlations giving rise to the $\nu = 1/2$ non-Fermi liquid state. (ii) Is disorder necessary for the possible IR equivalence of the two CF theories? (iii) Is PH symmetry emergent when the effects of Landau level mixing or PH symmetry-breaking disorder are considered? (iv) How does thermal Hall transport compare between the HLR and Dirac CF theories [21]?

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