A surprise in the physics of nonequilibrium interfaces

Strong coupling in conserved surface roughening: A new universality class?
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This Commentary draws attention to a recent paper that discovers an overlooked possibility, and associated richness, in the scaling of nonequilibrium interfaces. The underlying technical point is that currents associated with scalar conservation laws do not have to be gradient flows. The consequences appear to include a new phase governed by a strong-coupling fixed point. I summarize the paper below.

The height profile $h(\mathbf{x}, t)$ of an interface constrained to keep constant the volumes of the two domains it separates must obey a conservation law: $\partial_t h = -\nabla \cdot \mathbf{J}$, where in general the current $\mathbf{J}$ consists of a deterministic piece $\mathbf{J}^d$ and a Gaussian spatiotemporally white noise $\mathbf{J}^n$ encoding thermal or other random fluctuations. For an interface at thermal equilibrium $\mathbf{J}^d$ must act so as lower an interfacial energy $H \sim \int_{\mathbf{x}} \sqrt{1 + (\nabla h)^2}$, i.e., $\mathbf{J}^d \sim -\nabla \delta H/\delta h$, so that $\partial_t h \sim -\nabla^4 h$ with unimportant nonlinear corrections. If the interface is in a stationary state *far* from thermal equilibrium, then the mere fact that it separates two dissimilar media, i.e., that h and $-h$ are not equivalent, means that nonequilibrium terms of the form $(\nabla h)^2$ are permitted in the effective chemical potential whose gradients drive $\mathbf{J}^d$. Thus the h dynamics should read [1]

$$\partial_t h = -\nu \nabla^4 h - \frac{\lambda}{2} \nabla^2 (\nabla h)^2 + \eta \tag{1}$$

where the noise $\eta = \nabla \cdot \mathbf{J}^n$ satisfies $\eta(\mathbf{0}, 0)\eta(\mathbf{x}, t) = -2D\nabla^2 \delta(\mathbf{x})\delta(t)$, so that its strength vanishes as q^2 for wavenumber $q \to 0$. The Sun-Guo-Grant [1, 2] equation (1), a conserving variant of the KPZ equation [3], has generally been understood to be the universal description of the dynamics of a nonequilibrium interface, without inversion symmetry along its normal, that conserves the volume beneath it. Structure functions in d space dimensions are found to scale as $\langle |h(\mathbf{0}, 0) - h(\mathbf{x}, 0)| \rangle \sim |\mathbf{x}|^{2\chi}$ and $\langle |h(\mathbf{0}, 0) - h(\mathbf{0}, t)| \rangle \sim t^{2\chi/z}$ with spatial and temporal scaling exponents $\chi = \epsilon/3$ and $z = 4 - \epsilon/3$ at first order in $\epsilon = 2 - d$ (see [2]).

A hidden assumption, however, underlies the above discussion, as Caballero *et al.* point out. The invariance $h \to h+$ constant implies only that $\mathbf{J}$ must be *built* from gradients of

h; the curl of $\mathbf{J}$ need not be zero. The conserved-KPZ term in (1) arises from a gradient flow $\mathbf{J}_\lambda = (\lambda/2)\nabla(\nabla h)^2$, but a current of the form $\mathbf{J}_\zeta = \zeta\nabla h \nabla^2 h$ (see also [4]), whose curl is non-vanishing, is permitted at the same order in ∇ and h. Importantly, although $\mathbf{J}_\zeta$ can be decomposed into irrotational and solenoidal parts, with only the former participating in the equation of motion for h, the irrotational part of $\mathbf{J}_\zeta$ is distinct from $\mathbf{J}_\lambda$. The authors offer geometrical grounds for the existence of $\mathbf{J}_2$. It is potentially useful to view it as arising from a current proportional to a force density that is the divergence of a stress $\nabla_i h \nabla_j h$.

A perturbative one-loop dynamical renormalization-group (RG) analysis of (1) modified by $\mathbf{J}_\zeta$ reveals runaway flows for $1 < d \leq 2$, with the case $d = 2$ being obviously of greatest interest. The question is whether this is new physics or a pathology of the calculation. The authors present the results of a direct numerical solution of the stochastic PDE which, for $d = 2$, in the regime where the RG shows a runaway, displays strong growth of the height variance at long times. They also find that localized peaks appear after a waiting period and then coarsen. This makes a pretty convincing case for a new phase and scaling phenomena in the runaway regime. The observations presented suggest an intriguing connection to mound formation and related phenomena discussed in related models [5].

References

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