Return of the Hexatic : A Quantum Vortex Fluid?

1. **Melting of the Vortex Lattice through Intermediate Hexatic Fluid in an a-MoGe Thin Film**
   Authors: I. Roy, S. Dutta, A. N. R. Choudhury, S. Basistha, I. Maccari, S. Mandal, J. Jesudasan, V. Bagwe, C. Castellani, L. Benfatto, and P. Raychaudhuri
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2. **Evidence of Quantum Vortex Fluid in the Mixed State of a Very Weakly Pinned a-MoGe Thin Film**
   Authors: S. Dutta, I. Roy, S. Mandal, S. Basistha, J. Jesudasan, V. Bagwe, and P. Raychaudhuri
   arXiv:1905.01045

Recommended with a Commentary by Daniel Arovas, University of California, San Diego

Crystalline phases of matter which are characterized by a spontaneous breaking of continuous translational symmetry are forbidden in one and two space dimensions at any finite temperature as a consequence of the Hohenberg-Mermin-Wagner (HMW) theorem. The theory of 2D melting, developed over 40 years ago by Kosterlitz, Thouless, Halperin, Nelson, and Young (KTHNY), describes a two-stage process. In the crystal phase, while the spatial correlator $C_{\boldsymbol{G}}(\boldsymbol{R}) = \langle \varrho_{\boldsymbol{G}}(\boldsymbol{R})\, \varrho_{-\boldsymbol{G}}(\boldsymbol{0})\rangle \propto |\boldsymbol{R}|^{-\eta_G}$ decays to zero as $|\boldsymbol{R}| \to \infty$ in accordance with the strictures of HMW[†], its power law behavior characterizes a distinct 2D phase of matter, albeit one without a conventional Landau-type order parameter. The first stage of melting proceeds via a Kosterlitz-Thouless (KT) like transition into a phase in which thermally activated dislocations are unbound[‡]. This phase, known as the *hexatic*, is an anisotropic fluid exhibiting exponentially decaying positional correlations, but power law

[†]Here $\varrho_{\boldsymbol{G}}(\boldsymbol{R}) = e^{i\boldsymbol{G}\cdot\boldsymbol{u}(\boldsymbol{R})}$ is a Fourier component of the density, with $\boldsymbol{G}$ a reciprocal lattice vector and $\boldsymbol{u}(\boldsymbol{R})$ the local deviation from lattice site $\boldsymbol{R}$.

[‡]The topological defects which unbind when the crystal melts each carry a *vector charge* corresponding to their Burgers vector. This distinguishes the first stage of 2D melting from a conventional KT transition, where the topological defects carry a scalar charge. Another distinction *vis-a-vis* standard KT theory is that there are multiple stiffnesses which can be defined, and their discontinuity at the transition is not of the universal form $\Delta\rho_{\rm s} = \frac{2}{\pi} k_{\rm B} T_{\rm c}$.

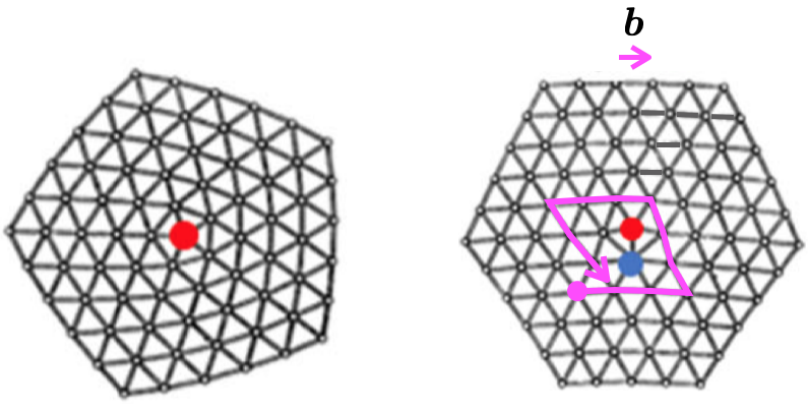

Figure 1: Left: A hexagonal lattice disclination of charge -1 (five rather than six nearest neighbors). Right: A dislocation composed of two oppositely charged disclinations, with Burgers vector $\boldsymbol{b}$. Adapted from ref. [2].

orientational order in the quantity $\psi_6(\boldsymbol{r}) = \nu_{\boldsymbol{r}}^{-1} \sum_{\boldsymbol{r}'}' \exp(6i\theta_{\boldsymbol{r}\boldsymbol{r}'})$, where $\theta_{\boldsymbol{r}\boldsymbol{r}'}$ is the angle made by the link between particle at $\boldsymbol{r}$ and one of its neighbors[†] at $\boldsymbol{r}'$ with respect to an arbitrary fixed direction in space, and $\nu_{\boldsymbol{r}}$ is the number of nearest neighbors of $\boldsymbol{r}$. The shear modulus is finite in the crystal but vanishes in the hexatic[‡]. A dislocation is equivalent to a bound pair (*i.e.* a dipole) of *disclinations* (see Fig. 1), and the second transition, from the hexatic to an isotropic fluid, is associated with the unbinding of disclination pairs and is of the standard KT class[§]. In the isotropic fluid, both $\varrho_{\boldsymbol{G}}(\boldsymbol{R})$ as well as $\psi_6(\boldsymbol{r})$ exhibit exponentially decaying correlations. 2D melting is not guaranteed to proceed via the two-stage KTHNY scenario, however. Depending on microscopic details, there could be a single first order transition between crystal and liquid, as in 3D. Or there could be a KT-like transition from crystal to hexatic and a first order transition from hexatic to isotropic fluid, as has been observed in numerical simulations of hard disks [1].

A natural setting in which to search for the hexatic is the vortex phase of superconducting sheets or films[¶]. In such quasi-2D structures, the sample thickness can be much smaller than the scale on which vortex lines bend, in which case the vortices behave as point particles. Some 30 years ago, using the Bitter pattern technique of decorating magnetic flux lattices with magnetic 'smoke' consisting of small magnetic particles, Murray *et al.* reported the observation of a "hexatic vortex glass" phase[‖] in BSCCO [4]. Another imaging method

[†]For a general configuration the nearest neighbors of a given site may be determined by a Delaunay triangulation.

[‡]In contrast to familiar δ-function peaks, the structure factor $S(\boldsymbol{G}+\boldsymbol{q}) = \sum_{\boldsymbol{R}} C_{\boldsymbol{G}}(\boldsymbol{R})\, e^{i\boldsymbol{q}\cdot\boldsymbol{R}}$ in the vicinity of the Bragg vectors exhibits a power law behavior $|\boldsymbol{q}|^{-2+\eta_G}$ as $\boldsymbol{q} \to 0$ in the crystal phase. In a *three*-dimensional hexatic, where the order parameter Ψ_6 is finite and independent of system size, there are Bragg "bumps" in $S(\boldsymbol{G}+\boldsymbol{q})$. For a 2D hexatic, the amplitude of these bumps vanishes in the thermodynamic limit.

[§]Disclinations carry scalar 'charges', as opposed to dislocations which carry (Burgers) vector 'charges'.

[¶]Hexatics have also been observed in two-dimensional colloidal crystals [3].

[‖]The putative hexatic glass phase is distinguished from the hexatic liquid due to the presence of quenched disorder which pins dislocations and results in a finite shear modulus. However, theoretical and numerical work has called to question the stability of this notional phase. Rather, disorder is expected to lead to a proliferation of disclinations resulting in no distinct hexatic glass phase in 2D.

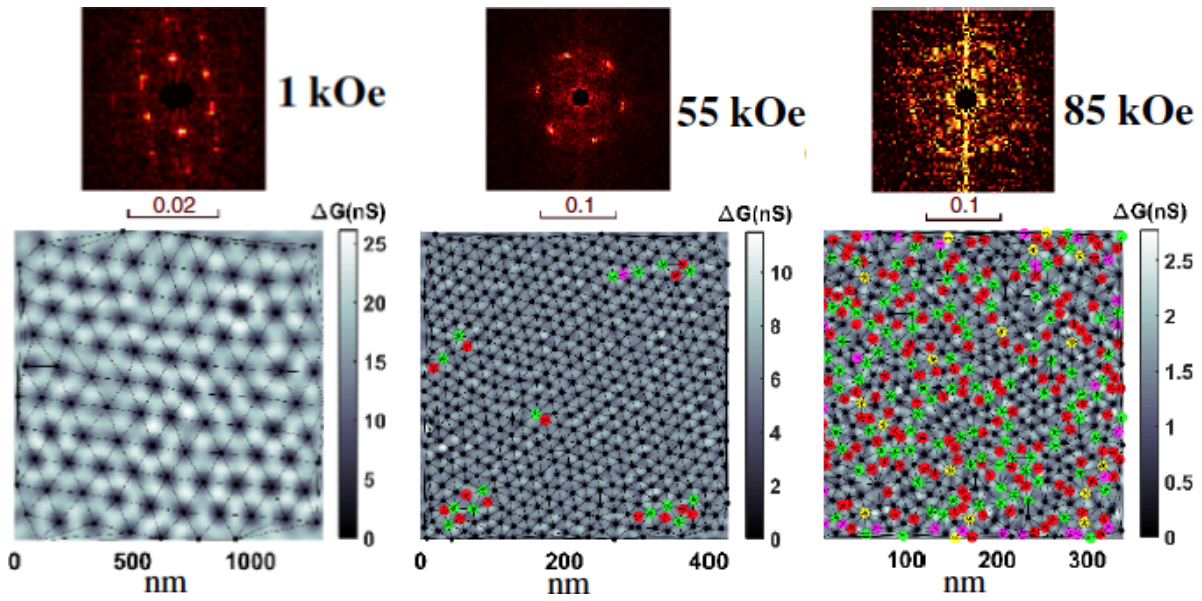

Figure 2: $T = 2$K images for three different magnetic fields [6]. Vortices (black dots) appear as minima in the STS conductance map, obtained at fixed dc bias $V_b = 1.52\,\mathrm{mV}$. Five- (red), seven- (green), four- (magenta), and eight-fold (yellow) coordinated topological defects are identified from the Delaunay triangulation of the vortex positions. Above each STS image is its 2D Fourier transform; the scale bar is in nm^{-1}. Note that the structure factor for the hexatic still exhibits prominent Bragg peaks, which presumably are a finite size effect.

maps the vortex positions via scanning tunneling spectroscopy (STS), taking advantage of the fact that the vortex cores are essentially metallic, and provide larger tunneling resistance than the superconducting interstices of the vortex matter [5].

The recent work of Roy *et al.* [6] combines real space STS imaging and transport measurements to map out the phase diagram of vortex matter in very weakly pinned, amorphous MoGe thin films ($t \approx 20\,\mathrm{nm}$, $T_c \approx 7.05\,\mathrm{K}$). The pinning strength is estimated to be six orders of magnitude weaker than corresponding values in Nb or $YBa_2Cu_3O_7$ films. The results are in substantial agreement with the KTHNY scenario. The crystal phase is characterized by an activated resistance $R(I) = R_{\mathrm{ff}} \exp[-U(I)/k_B T]$, where $U(I) = U_0(I_c/I)^\alpha$ with I_c the depinning current. For $I > I_c$ the vortex crystal is in a flux flow regime where the voltage drop generated by vortex motion is $V = R_{\mathrm{ff}}\,(I - I_c)^\dagger$. For $I < I_c$ they find $\alpha = 1$ provides a very good fit. Thus $R \to 0$ exponentially in the crystal. At fixed T, increasing magnetic field yields a transition into a vortex fluid state at a characteristic field $H = H^1_{\mathrm{M}}(T)$. In this fluid phase, $U(I)$ and thus $R(I)$ are independent of current as $I \to 0$. That the resistivity $\rho(H > H^1_{\mathrm{M,max}})$ remains finite in the $T \to 0$ limit suggests that the vortices in this regime may be in a quantum fluid state [8]. The identification of the vortex fluid as a hexatic phase is established by STS imaging of the vortex configurations. At $T = 2\,\mathrm{K}$ and $H = 1\,\mathrm{kOe}$, a hexagonal vortex lattice is observed with no defects. Above $H = 3\,\mathrm{kOe}$, free dislocations in

[†]It should be stressed that there is a rich panoply of dynamical phases of interacting vortex matter [7], and so the dichotomy between pinned and Ohmic phases is something of a cartoonish view. Nevertheless, for $I \gg I_c$ one expects Ohmic behavior.

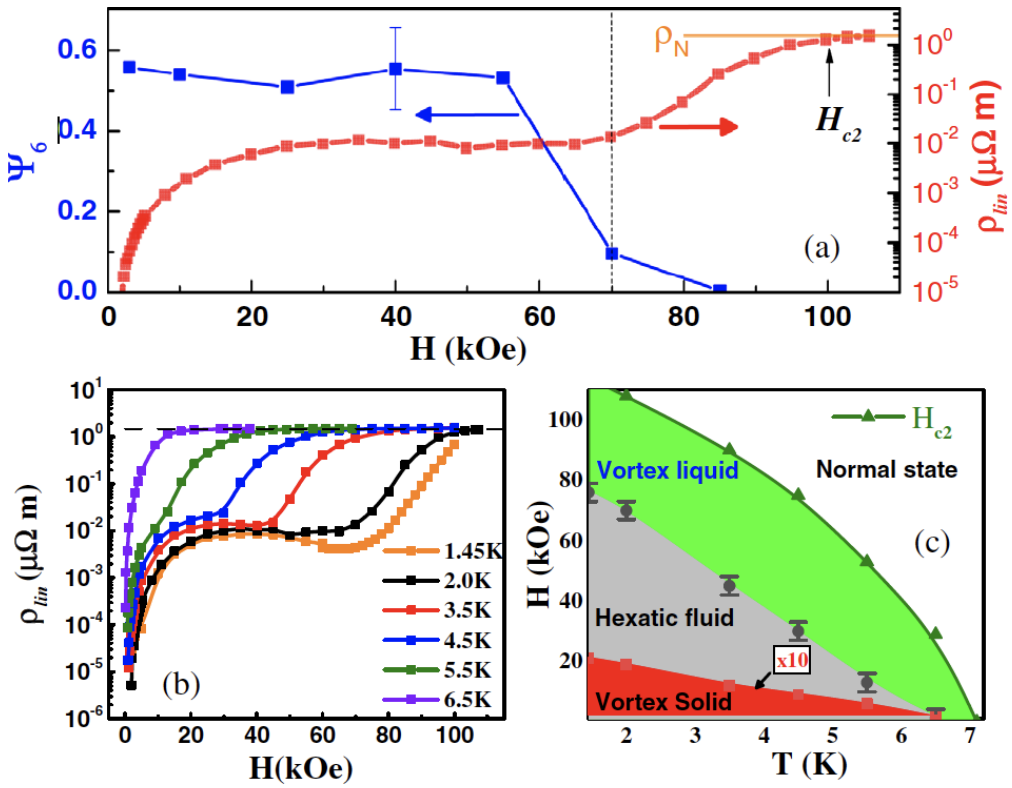

Figure 3: Top: Hexatic order parameter Ψ_6 and linear resistivity $\rho_{\rm lin}$ as a function of field at $T = 2\,{\rm K}$. Lower left: $\rho_{\rm lin}(H,T)$ *versus* H at different temperatures. Lower right: Phase diagram inferred from transport and STS measurements. The mean field transition labeled by H_{c2} is merely a crossover, and there is no essential distinction between the isotropic vortex liquid and the normal state. From ref. [6].

the vortex lattice are present, but for $H < H_{\rm M}^2 = 70\,{\rm kOe}$ the Fourier transform of the STS conductance pattern $G(\boldsymbol{r})$ exhibits six peaks, corresponding to sixfold orientational order (see Fig. 2). At $H = H_{\rm M}^2$, the six peaks spread into a ring, heralding an isotropic vortex liquid phase. The phase boundaries are corroborated by dynamical studies of the un-driven vortex motion (likely induced by thermal and perhaps quantum fluctuations) gleaned from 12 successive images over the same spatial field at 15 minute intervals. In the crystal phase, each vortex undergoes small excursions about its mean position. In the high field liquid phase, the vortex motions are random. In the hexatic fluid, however, the long time motion of the vortices is along one of the three principal directions of the hexagonal structure, consistent with the gliding of dislocations [2].

Orientational order in the putative hexatic fluid is computed by evaluating the order parameter $\Psi_6 \equiv N^{-1} \sum_{\boldsymbol{r}} \psi_6(\boldsymbol{r})$, where N is the number of vortices[†]. For a perfect hexagonal lattice, $\Psi_6 = 1$. Roy *et al.* observe that Ψ_6 remains roughly constant throughout the hexatic phase as H is varied at fixed system size, and then abruptly plunges to zero in the

[†]More precisely, one defines the correlation function $g_6(\boldsymbol{r}) = \langle \psi_6(\boldsymbol{r})\, \psi_6(\boldsymbol{0}) \rangle \propto |\boldsymbol{r}|^{-\eta_6}$. One then defines $\Psi_6 \equiv N^{-1} \big\langle \sum_{\boldsymbol{r},\boldsymbol{r}'} \psi_6(\boldsymbol{r})\, \psi_6^*(\boldsymbol{r}') \big\rangle^{1/2} \propto L^{-\eta_6/2}$, where $L \propto N^{1/2}$ is the characteristic linear dimension

isotropic fluid at $H = H_{\mathsf{M}}^2$. Concomitant with this feature, the resistivity ρ_{lin}, computed by averaging $\langle dV/dI \rangle$ over the interval $I \in [0, 100\,\mu\mathrm{A}]$, begins to rise sharply and continues to do so throughout the isotropic vortex liquid phase until it achieves its normal state value. Curiously, at the lowest temperatures $\rho_{\mathsf{lin}}(H)$ exhibits a local minimum at the boundary between the hexatic fluid and the isotropic liquid (*i.e.* at $H = H_{\mathsf{M}}^2$), whereas for $T \gtrsim 2\,\mathrm{K}$, $\rho_{\mathsf{lin}}(H)$ is monotonic and has a kink at $H = H_{\mathsf{M}}^2$, with an apparent slope discontinuity.

The observations in these papers provide a beautiful example of the KTHNY scenario for 2D melting. It would be worthwhile to investigate the system size dependence of the hexatic order parameter Ψ_6 to extract the exponent η_6, or to obtain the correlator $\langle \psi_6(\boldsymbol{r})\, \psi_6(\boldsymbol{r}') \rangle$ in this phase. Whether the $T \to 0$ state of the hexatic is indeed a quantum vortex fluid (QVF) is a fascinating question. Dutta *et al.* [8] adduce further evidence for a QVF from the STS measurements at $T = 0.45\,\mathrm{K}$, which show that a soft gap in the local tunneling density of states (LTDOS) persists to the vortex core center. This is somewhat in contrast to what one expects from the standard Caroli-de Gennes-Matricon theory of fermionic states in superconducting vortex cores, where the LTDOS is expected to be flat or to have a small zero bias peak, and may reflect zero-point fluctuations of the vortex positions. It will also be interesting to characterize the crystal, hexatic, and isotropic liquid phases by AC measurements, and by other transport signatures such as the Nernst coefficient; work in this direction is apparently underway.

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