Viscoelasticity Cannot Create Stability

1. **Delayed instabilities in viscoelastic solids through a metric description**
   Authors: E. Urbach and E. Efrati
   arXiv:1711.09491

2. **The metric description of viscoelasticity and instabilities in viscoelastic solids**
   Authors: E. Urbach and E. Efrati
   arXiv:1806.01237

Recommended with a Commentary by Sam Dillavou and Shmuel Rubinstein
Harvard University, Cambridge MA, USA

From colloidal gels [1] to tectonic faults [2], many systems exhibit the memory effect of delayed failure. In many systems, this phenomenon is clearly associated with stress corrosion induced by chemistry and other irreversible processes [3-4] . However, in some cases [1-2, 5] the source of the history-dependent failure is less clear and may stem form configurational and visco-elastic effects. A robust and flexible framework that describes the mechanics of viscoelasticity has yet to emerge, and often as a result, one resorts to computational models to describe these systems. In two recent pre-prints, Urbach and Efrati propose an intuitive geometrical description of viscoelasticity, in which a continually evolving metric tensor dictates the instantaneous local lowest energy state of a material. Within this geometrical framework, the question of the viscoelastic origin for delayed stability received a surprising but definitive answer.

Consider a simple viscoelastic system like a rubber band starting from a relaxed state. Stretch the band in a single direction and release it immediately, and it will have the same elastic rest length, L_0, as before. Stretch it again in the same way, and the stress-strain response will be identical. Now stretch the band and hold it at constant length L for an extended time t, and three things occur. First, the stress exerted by the band decreases over time. Second, when released, the band will snap back to an intermediate length L_1, somewhere between L_0 and L. Finally, under no stress, the band will slowly shrink until it reaches its original rest length L_0. Urbach and Efrati describe this trio of effects in a framework of changing rest-lengths (metrics.) The lowest elastic energy state of the band, that is, the instantaneous state the band wants to snap to, is given by $\bar{L}(t)$. The evolution equation for $\bar{L}$ contains two terms, one pulling $\bar{L}$ towards the current length of the band, L, and one pulling towards the original rest length L_0. Conceptually, it is now an easy jump to a 3D continua, e.g. a block of memory foam, in which the evolving rest length is generalized by an evolving rest metric $\overline{g_{ij}}(t)$, that is 'the metric on which the body is locally stress-free and stationary.' The strain tensor of the body is calculated as $\varepsilon_{ij}(t) = \frac{1}{2}(g_{ij}(t) -$

$\overline{g_{ij}^0})$, where $\overline{g_{ij}^0}$ is the instantaneous rest metric, and $g_{ij}(t)$ is the current configuration of the system. Here too the rest metric evolves toward two values, g_{ij}^0 and $g_{ij}(t)$.

The tidiness of this framework shines through most in its treatment of stability. In one dimension, these co-evolutions of $\bar{L}$ towards both L_0 and L mean that the equilibrium position for $\bar{L}$ will lie at some value $L_0 < \bar{L} < L$. In two dimensions or above, the corollaries are less trivial; because of the two terms in the evolution equation, any equilibrium position for vector $\bar{L}$ must be colinear with L and L_0. Urbach and Efrati show a surprising result of this rule for incompressible viscoelastic materials in the linear regime in any dimension; no permanently stable states for L may be created through the evolution of the instantaneous rest length $\bar{L}$, and conversely, a stable state when the system is relaxed will never lose stability due to viscoelasticity. As a demonstration of this principle, Urbach and Efrati utilize a model system: conic silicone-rubber shells of varying thicknesses. The thinnest shells are bi-stable – they hold their shape when inverted inside-out. In contrast to the thickest shells, which immediately snap back, inverting to their original shape. Shells of medium thickness snap back immediately when inverted, unless held in their inverted state for a period before release. That is, bi-stability is created through evolution of the system's rest metric. However, none of these created bi-stable states is permanent, although stability lasts for minutes if held long enough. This is the crux of the matter: viscoelasticity can delay the instability, but it cannot eliminate it.

The framework presented by Urbach and Efrati is mathematically elegant and clean and is thus a fitting tool to determine what general behaviors visco-elastic systems are capable of. For example, is there a point in the tectonic cycle past which an earthquake is inevitable, but the system is temporarily stable? The framework may also provide a backbone for the description of many complex systems of current interest. Several systems that have long been known to demonstrate viscoelastic behavior, such as frictional interfaces [6-7] and crumpled paper [8], have recently been shown to display a more complex memory than previously thought [9-10]. The capability of these systems, for example, to produce a non-monotonic evolution in stress under constant strain is also possible in the framework discussed here. Time will tell what complex and interesting systems this framework can describe.

References

[1] SB Lindstrom et. al. *Structures, stresses, and fluctuations in the delayed failure of colloidal gels*. Soft Matter (2012)

[2] Omori, F. *On the after-shocks of earthquakes (Vol. 7)*. Imperial University of Tokyo. (1894)

[3] Wiederhorn, S.M. and Bolz, L.H. *Stress corrosion and static fatigue of glass*. Journal of the American Ceramic Society, 53(10), pp.543-548. (1970)

[4] Johnson, H.H., Morlet, J.G. and Troiano, A.R. *Hydrogen, crack initiation, and delayed failure in steel*. Trans. Met. Soc. AIME, 212. (1958)

[5] Freed, A.M. and Lin, J. *Delayed triggering of the 1999 Hector Mine earthquake by viscoelastic stress transfer*. Nature. (2001)

[6] E Rabinowicz, *Friction and Wear of Materials* Wiley New York (1965)

[7] JH Dieterich and BD Kilgore, *Direct Observation of Frictional Contacts*, Pure Appl Geophys (1994)

[8] K Matan et. al., *Crumpling a Thin Sheet.* Phys Rev Let (2002)
[9] S Dillavou and SM Rubinstein, *Nonmonotonic Aging and Memory in a Frictional Interface*. Phys Rev Let (2018)
[10] Y Lahini et. al., *Nonmonotonic Aging and Memory Retention in Disordered Mechanical Systems*. Phys Rev Let (2017)