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Four regimes of transport in a two-dimensional Fermi fluid

1. **The Hierarchy of Excitation Lifetimes in Two-Dimensional Fermi Gases**
   Authors: Patrick J. Ledwith, Haoyu Guo, Leonid Levitov
   arXiv:1905.03751
2. **Fermionic Retroreflection, Hole Jets and Magnetic Steering in 2D Electron Systems**
   Authors: Lev Haldar Kendrick, Patrick J. Ledwith, Andrey Shytov, Leonid Levitov
   arXiv:1810.07588
3. **Particle Collisions and Negative Nonlocal Response of Ballistic Electrons**
   Authors: Andrey Shytov, Jian Feng Kong, Gregory Falkovich, Leonid Levitov
   arXiv:1805.06819, Phys. Rev. Lett. **121** 176805 (2018)
4. **Fluidity Onset in Graphene**
   Authors: Denis A. Bandurin, Andrey V. Shytov, Leonid Levitov, Roshan Krishna Kumar, Alexey I. Berdyugin, Moshe Ben Shalom, Irina V. Grigorieva, Andre K. Geim, Gregory Falkovich
   arXiv:1806.03231, Nature Commun. **9** 4533 (2018)

Recommended with a Commentary by Francisco Guinea, Imdea Nanoscience, Madrid, Spain, and Donostia International Physics Center, San Sebastian, Spain

The electron gas is one of the main paradigms in condensed matter physics. The articles summarized in the commentary describe novel properties and a new regime in the two dimensional electron gas.

The concept of the electron gas is used in textbooks to illustrate the main features of many body systems, and it is the simplest model which describes a metal. In three dimensional metals which resemble closely an electron gas, such as Al, K or Na, noticeable effects of the lattice can be appreciated, such as anisotropies in the Fermi surface. Advances is semiconductor physics eventually provided a route to very perfect *two dimensional* electron gases, 2DEG[1]. In these systems, the confinement of electrons at surfaces lead to a narrow metallic layer with electron densities of order $n_e \sim 10^{12}\text{cm}^{-1}$. These densities imply distances between electrons about two orders of magnitude larger than the crystal lattice constant. Hence, the electronic wavelength in these systems is larger than the atomic spacing. Effects of the atomic lattice are averaged out, as well as impurities with a short range potential. The result is the formation of quite perfect electron gases. A further advance was the discovery of graphene, where similar electron densities, and similar degree of purity, was achieved[2].

Two dimensional electron gases have been extensively studied, both theoretically and experimentally. This research has led to amazing discoveries, such as the Quantum Hall Effect. Topological arguments show that new quasiparticles, such as anyons, or Majorana particles can exist in a strongly interacting two dimensional electron gas.

Despite the extensive work carried out during the last four decades, the two dimensional electron gas keeps bringing interesting surprises. One of them has been the characterization of the electronic hydrodynamic regime, postulated theoretically, but observed experimentally only recently[3]. In this regime, (quantum) electrons behave as a viscous fluid, and a description of the electronic properties based on weakly interacting quasiparticles breaks down.

The laws of hydrodynamics, like the Navier-Stokes equation, apply to systems of strongly interacting particles, where the viscosity describes the tendency of the particles to reach local equilibrium through mutual interactions. The Fermi-Dirac statistics followed by electrons imply that they interact weakly at low temperatures, and the electronic scattering time, τ_{ee}, in two dimensions, scale as $\hbar\tau_{ee}^{-1} \propto (k_B T)^2/E_F \log(E_F/T)$, where E_F is the Fermi energy. The hydrodynamic regime sets in when the mean free path due to electron-electron scattering, $\ell_{ee} \sim v_F \tau_{ee}$, where v_F is the Fermi velocity, is smaller than the dimensions of the system. Hence, the observation of the hydrodynamic regime implies not too low temperatures, and no other sources of scattering such as impurities and phonons. Graphene is an excellent platform where this regime can be studied, as it is quite free of impurities, and the lightness of the carbon ion implies that phonons are only activated at high temperatures.

The detailed characterization of the electronic hydrodynamic regime in graphene suggested that the transport properties of the two dimensional electron gas could be classified into three main classes: i) diffusive, dominated by scattering by impurities or phonons, with a mean free path smaller than the sample dimensions, and where interactions play a minor role, ii) ballistic, where the mean free path is larger than the sample dimensions, and where, again, interactions can be ignored, and iii) the hydrodynamic regime, dominated by interactions, and where scattering by impurities and phonons is negligible.

The theoretical work reported in[4, 5] suggests that there is an additional regime, where interactions modify in a subtle way ballistic electrons, but where they are not strong enough to give rise to hydrodynamic behavior.

The existence of this intermediate regime is due to a property of electron-electron collisions in two dimensions, which is not shared by the classical atoms or molecules which describe ordinary fluids. As mentioned earlier, electrons are described by the Fermi-Dirac distribution. At temperatures such that $k_B T \ll E_F$ the absolute value of their momenta is within a small range of values around $k_F = E_F/(\hbar v_F)$. Electron-electron collisions conserve energy and momentum, and, as a result, the phase space available for scattering processes shows a sharp maximum when the momenta of the electrons is nearly parallel. Electrons move in two dimensions, but most collisions are head on. This feature leads to the logarithmic correction of the electron-electron scattering time mentioned earlier.

In the ballistic regime, electron-electron collisions can lead to interesting consequences, even if ℓ_{ee} is larger than the dimensions of the sample. For instance, as shown in[4], ballistic electrons injected in such a two dimensional electron gas scatter backwards electrons already in the sample. As a result, the number of electrons moving towards the region where the current is injected is lower than in the absence of the current. This electron deficit is compensated by a negative non local voltage. This regime is a precursor to the hydrodynamic regime, which sets in when the collision rate increases, and ℓ_{ee} becomes shorter than the sample dimension. The existence of this regime was experimentally observed in[6].

The fact that most electron collisions in the two dimensional electron gas are either forward or backward scattering has other interesting consequences. The relaxation of the electron momenta does not follow a simple exponential dependence on time, as expected in a classical chaotic system. The backward scattering processes between injected electrons and electrons from the reservoir, mentioned in the previous paragraph, can be seen as the creation of forward moving holes, as in Andreev scattering processes in a superconductor. These "hole jets" can be quite well defined and narrow, leading to observable consequences in weak magnetic fields, as discussed in[5].

Last, but not less important, is the effect of forward and backward scattering processes on relaxation and thermalization of the electron gas when it is taken out of equilibrium. Collective electronic excitations can be described as deformations of the Fermi surface. In two dimensions, where the Fermi surface is a circle, these fluctuations can be expressed in terms of angular harmonics, with angular momentum m. Simple phase space arguments suggest that the relaxation rate of these fluctuations should be given by a relaxation time with a temperature dependence $\hbar\tau_m^{-1} \propto m^2(k_BT)^2/E_F$, up to logarithmic corrections. Reference[7] presents a very complete analysis of the collision integral of the two dimensional electron gas, and uses it to estimate the relaxation rate of excitations as function of angular momentum. The work makes use of the fact that $\alpha = (k_BT)/E_F \ll 1$, and uses the value of α as an expansion parameter[8].

The expected, Fermi liquid like, dependence on the square of the temperature is obtained only when the angular momentum, m, is even. If m is odd, backward scattering is suppressed, as electrons with momentum $+\vec{\mathbf{k}}$ are not compensated by electrons with momentum $-\vec{\mathbf{k}}$. Scattering can still occur, due to existence of thermally activated electrons, but the decay rate, if m is odd, is $\hbar\tau_m^{-1} \propto m^4(k_BT)^4/E_F^3$. The relaxation of the even m excitations can be described by a diffusion equation over angles (due to the m^2 factor in τ_m^{-1} mentioned earlier). The relaxation of the odd m excitations can be defined as "superdiffusion", as the required operator is ∂_θ^4, instead of ∂_θ^2, the operator in a more standard diffusion process. As a result, the electron gas remains out of equilibrium for long times, going through an intermediate state where some modes relax but others not. It is worth noting that the separation between head on and other collisions in a two dimensional electron system is rather general, and it can be extended to non circular Fermi surfaces, and it does not require a parabolic dispersion either.

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[7] Patrick J. Ledwith, Haoyu Guo, Leonid Levitov, **The Hierarchy of Excitation Lifetimes in Two-Dimensional Fermi Gases**, arXiv:1905.03751.

[8] Note that a different, scale invariant regime, can be observed in graphene and Weyl semimetals, when the Fermi energy lies near the Dirac point, and $(k_B T)/E_F \gg 1$. See, for instance: Lars Fritz, J´org Schmalian, Markus Mller, and Subir Sachdev, **Quantum critical transport in clean graphene**, Phys. Rev. B **78**, 085416 (2008), Alberto Cortijo, Yago Ferreirós, Karl Landsteiner, Mara A. H. Vozmediano, **Hall viscosity from elastic gauge fields in Dirac crystals**, 2D Materials **3**, 1 (2016) .