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Fragility in time-reversal protected topological phases

Time's Arrow and the Fragility of Topological Phases
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Recommended with a Commentary by Michael Zaletel, University of California, Berkeley

In addition to their formal elegance, much of the appeal of topological phases lies in their robustness: the combination of non-trivial quantum entanglement, a bulk energy gap E_g, and a finite correlation length ξ, leads to a variety of universal features, such as edge states, zero-modes, and fractionalized quasiparticle statistics, which are insensitive to the details of the Hamiltonian. Ideally, one might hope that this robustness would imply that all the "useful" properties of such phases, such as their potential to store quantum information or transport charge, would be protected up to *exponentially* small corrections in any experimental parameter such as temperature, sample size, the distance between quasiparticles, the timescale of measurements, etc. However, depending on the property in question, this need not be the case, and going beyond toy models to determine the precise scope of topological robustness - and observing its effect in experiment - is an ongoing challenge.

At the outset it is important to distinguish between two very different classes of topological phase. The first class are the "intrinsic" topological phases, such as the fractional quantum Hall effect, which feature quasiparticles with fractional statistics. These phases exist irrespective of any symmetry, and in certain cases they feature non-Abelian excitations which may form the basis for decoherence-resistant qubits. The second class - and the subject of McGinley and Cooper's analysis - are the "symmetry protected" topological (SPT) phases, such as the $\mathbb{Z}_2$-topological insulator or the Haldane (AKLT) phase of the 1D $S = 1$ Heisenberg antiferromagnet. These phases do not have fractionalized quasiparticles, but do feature gapless edge states which are "protected" so long as the symmetry is preserved. The 1D $S = 1$ Heisenberg antiferromagnet, for example, has a gap to bulk excitations (the Haldane gap) but features $S = 1/2$ edge states protected either by time-reversal or spin-rotation symmetry. The edge state itself is not particularly exotic - it behaves just like an isolated spin-1/2 moment. The novelty is rather that it emerges in a system composed of *integer* spins. This explains its robustness: there is no way for the $S = 1/2$ edge moment to gap out through hybridization with the bulk, since the fusion of integer and half-integer spin will always leave a half behind.

In the mathematical treatment of symmetry protected topological phases, anti-unitary symmetries like time-reversal $\mathcal{T}$ can be put on more-or-less the same footing as unitary symmetries like charge conservation or spin rotation, with some complex-conjugations sprinkled into the definitions. It is then tempting to assume that the phenomenology of the unitary and $\mathcal{T}$-protected edge states is analogous as well. McGinley and Cooper identify a situation where this is not the case: in the presence of a bath (albeit with an important caveat mentioned below), the coherence time τ of a unitarily-protected 0D edge state will diverge exponentially with inverse-temperature, while for an anti-unitary symmetry it diverges only as a power law ("fragility"). This qualitative distinction arises from an essential difference in how locality manifests in unitary and anti-unitary symmetries. The action of a local unitary symmetry g factorizes as a tensor product across the system and bath, $\hat{g} = \hat{g}_S \otimes \hat{g}_B$, but for an anti-unitary symmetry, $\hat{g} = \hat{g}_S \otimes \hat{g}_B \mathcal{K}$, there is no sense in which the complex conjugation $\mathcal{K}$ factorizes. Consequently, while it is clearly meaningful to demand that a system-bath interaction H_{SB} preserve a unitary symmetry *independently* in S and B, this is not the case for $\mathcal{T}$, which appears to be the origin of their differing behavior.

To analyze the problem, they consider a symmetry preserving system-bath Hamiltonian $H = H_S + H_B + H_{SB}$ where the coupling takes the general form $H_{SB} = \sum_\alpha A_\alpha B_\alpha$. There are two different "rules of the game" we might impose here: either the A_α, B_α are individually symmetric, or only their product is. The latter case is the most generic, and arguably the more physical; for example, the edge state of a spin-chain could interact with the bath through a Heisenberg coupling, $\mathbf{S}_E \cdot \mathbf{S}_B$. However, such a coupling will generically decohere the edge state irrespective of the bulk gap, since it's really no different than any other $S = 1/2$ moment which will acquire a finite $T_{1/2}$. So, to give the edge a fighting chance, they restrict to the situation where A_α, B_α are *independently* symmetric; in the unitary case, this is equivalent to requiring that H is symmetric under $\hat{g}_S, \hat{g}_B$ independently. It is in this restricted scenario that a surprising difference between time-reversal and unitary-protected edge states becomes manifest.

Their main technical result follows from tracing out the bath and analyzing the implications of the microscopic symmetries on the structure of the resulting master equation (Lindbladian) for the 1D system. Interestingly, to see the effect it is necessary to go to fourth-order in the system-bath coupling H_{SB}, while the most standard treatment goes to second-order. However, as they explain, the gist of the effect can be seen from the structure of time-dependent perturbation theory. Consider starting in a state where the system and bath are un-entangled, $|\psi\rangle = |S\rangle \otimes |B\rangle$, with $|S\rangle$ a state in the degenerate ground state manifold, which forms an irreducible representation of the symmetry group, and time-evolve under the perturbation H_{SB}. If we focus on the part of the evolution which isn't suppressed by $e^{-\beta E_g}$, to first-order in the coupling the state will acquire a component proportional to $|\delta\Psi\rangle \ni \Pi_{\mathrm{GS}} A_\alpha |S\rangle \otimes B_\alpha |B\rangle$, where Π_{GS} projects back onto the ground-state manifold of the system. Since the edge states form an irrep, Schur's Lemma tells us that $\hat{A}_\alpha$ must act as the identity when projected into the ground-state manifold, e.g., $\Pi_{\mathrm{GS}} \hat{A}_\alpha |S\rangle = a_\alpha |S\rangle$ for some number a_α. Consequently, the subsequent evolution of the bath is not entangled with the state of the edge, which, after tracing out the bath, implies the absence of decoherence to leading order.

However, the situation is quite different if we continue to second-order, $|\delta^2\Psi\rangle \ni \Pi_{\mathrm{GS}} A_\alpha \Pi_{\mathrm{ex}} A_\beta |S\rangle \otimes B_\alpha B_\beta |B\rangle$. Defining jump operators $C_{\alpha\beta} \equiv \Pi_{\mathrm{GS}} A_\alpha \Pi_{\mathrm{ex}} A_\beta \Pi_{\mathrm{GS}}$, which are symmetric, it might seem we can again appeal to Schur's Lemma. However, $C_{\alpha\beta}$ need not be Hermitian: $C^\dagger_{\alpha\beta} = C_{\beta\alpha}$. In the anti-unitary case, Schur's Lemma applies only to Hermitian operators, an obvious example being $O = iS^z$: O is preserved under time-reversal but acts non-trivially on spin. So to proceed we first decompose C into it's Hermitian and anti-Hermitian components, $C_{\alpha\beta} = X_{\alpha\beta} + iY_{\alpha\beta}$, where X, Y are now both Hermitian. For a unitary symmetry, X, Y will be independently symmetric and Schur's Lemma applies to each, so the coupling acts trivially and decoherence is suppressed. For an anti-unitary symmetry, however, the "i" spoils this argument and Y will break time-reversal. Y can then act non-trivially on the ground-state manifold, entangling the system with the bath and decohering the edge. They construct some simple system-bath couplings which manifest this effect. The resulting coherence time τ depends on the details of the bath, but for a gapless bath it will generically scale as a power-law in with inverse temperature; they find an ohmic bath, for example, gives $\tau \sim \left(\frac{E_g}{V}\right)^4 \frac{\omega^4}{T^5}$, where V is the scale of the system-bath coupling, T the temperature, and ω_c is characteristic scale of the bath.

While motivated by the edge states of 1D SPT phases, the bulk plays no essential role in their analysis, which applies just as well to any local degree of freedom which transforms under a multi-dimensional irrep of the symmetry group. Yet it is interesting to speculate how it may generalize to higher dimensional SPT phases like the 2D and 3D topological insulators. It is already known that the vaunted helical edge-states of the 2D $\mathbb{Z}_2$ topological insulator, which are protected from single-particle backscattering off elastic $\mathcal{T}$-symmetric impurities, develop a power-law in T resistance in the presence of a bath. In fact, a bath isn't required at all - with interactions, two-particle back scattering becomes allowed. Furthermore, in contrast to 1D where unitary / anti-unitary symmetry leads to an exponential / power-law in T coherence time under the restricted type of coupling, an isolated 2D TI with axial symmetry (a unitary S^z rotation) already admits $R \sim T^{-6}$ resistance. Nevertheless, it would be interesting to revaluate these works from their point of view.

It should also be noted that these considerations do not figure into to the coherence time of quantum information stored in an intrinsic topological order, such as the non-Abelian excitations of the $\nu = \frac{5}{2}$ fractional quantum Hall state. In these phases, the information is encoded in non-local degrees of freedom which can't be measured by a local coupling to the environment, irrespective of symmetry (which isn't to say the presence of a gapless bath, like photons, won't raise other issues). As XG Wen has joked, SPT can alternatively stand for "symmetry protected trivial" phases, and this hierarchy of entanglement may yet play out in the scope of their robustness as well.

References

[1] Time's Arrow and the Fragility of Topological Phases, Max McGinley and Nigel R. Cooper, arXiv:2003.08120 (2020).