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How universal is Hydrodynamics?

Non-Hydrodynamic Initial Conditions are Not Soon Forgotten
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Recommended with a Commentary by Jörg Schmalian, Karlsruhe Institute of Technology

Hydrodynamics is a theory of formidable power. When a classical or quantum fluid is dominated by strong collisions between its constituents and usual, perturbative approaches fail, robust phenomenological statements about the long-time dynamics can still be made with the help of hydrodynamic reasoning[1, 2, 3, 4]. The three pillars of the theory are: i) local equilibration, ii) the dynamics is determined by densities of conserved quantities and slowly varying order parameters, and iii) the validity of constitutive relations. A classical example is diffusion. Fick's law

$$\boldsymbol{j} = -D\nabla\rho \tag{1}$$

between the particle current and a density gradient is combined with particle conservation, expressed in terms of the continuity equation

$$\partial_t\rho + \nabla\cdot\boldsymbol{j} = 0. \tag{2}$$

Together this determines the long-time dynamics of the Fourier modes of the density

$$\rho\left(\boldsymbol{k},t\right) = \rho\left(\boldsymbol{k},0\right)e^{-t/\tau(k)}, \tag{3}$$

where the time scale $\tau\left(k\right) = \frac{1}{Dk^2}$ diverges in the long-wavelength limit in a well defined fashion.

Starting from a more microscopic kinetic theory, hydrodynamics can be shown to correspond to a description in terms of appropriately weighted first moments of some underlying single-particle distribution function. Higher moments correspond to fluctuations, often captured in terms of stochastic kinetic equations such as the Boltzmann-Langevin approach[5]. Within *fluctuation hydrodynamics* one encounters long-time tails of these higher moments as a generic feature. A local density fluctuation is naturally correlated to a local current fluctuation through Eq.1, leading to non-analytic behavior in the low-frequency power spectrum of current fluctuations of the form $\left|\omega\right|^{\frac{d-2}{2}}$ with d the space dimension[6, 7]. After a time span governed by the above exponential decay, current fluctuations eventually decay according to

$$\left\langle \boldsymbol{j}\left(t\right)\cdot\boldsymbol{j}\left(0\right)\right\rangle \propto t^{-d/2}. \tag{4}$$

In view of these considerations, Kirkpatrick, Belitz, and Dorfman address in their paper a fundamental question: Is the long-time dynamics of a first moment of the distribution function, such as the particle density, universal if one includes long-time tails due to higher moments? Even though long-time tails are by themselves insensitive to the microscopic details, the answer obtained by Kirkpatrick *et al.* is, surprisingly, no. The authors show that Fourier modes of the density $\rho(\boldsymbol{k},t)$ of a system with long-time tails are affected by initial condition effects that are not related to properties of the conserved quantity alone. This is obviously in contrast to the expectation of Eq.3. The specific problem studied in the paper is diffusion in a disordered electron system, where the long-time tails have their origin in weak-localization effects. The latter are incorporated in a kinetic Boltzmann theory via a time dependent diffusion constant $D \to D(t-t')$[8]. The specific result for $t \gg \tau(k)$ is

$$\delta\rho(\boldsymbol{k},t) = \alpha\left(\delta\rho(\boldsymbol{k},0)\left(\frac{\tau(k)}{t}\right)^{3/2} + \frac{3}{2}\delta\rho_{\perp}(\boldsymbol{k},0)\left(\frac{\tau(k)}{t}\right)^{5/2}\right), \tag{5}$$

where $\alpha = 3/\left(4\sqrt{\pi}k_F l_{\mathrm{mfp}}\right) \ll 1$. Here $\delta\rho_{\perp}(\boldsymbol{k},0)$ contains non-hydrodynamic initial conditions, due to observables other than the density itself. The analysis that led to this result suggests that the conclusions are more general and extend to generic systems governed by hydrodynamic equations. In addition, one expects that for systems with strong collisions, the analog to the coefficient α is no longer small.

The investigation of non-hydrodynamic effects within a hydrodynamic description is, of course, not new. Water that flows through a pipe behaves near the pipe's walls in a way that is not governed by hydrodynamics. However, the impact of this Knudsen layer can be incorporated in terms of boundary conditions for the fluid velocity through a slip length l_s, a concept that goes back to Maxwell[9]. In a way l_s is its own kinetic coefficient which for very rough walls is of the order of the momentum-conserving mean free path. It takes into account that within a few mean free paths away from the wall insufficient particle-particle collisions took place to wipe out the memory of scatterings at the wall. The authors discuss that for systems without long-time tails the microscopic initial conditions can be incorporated through a corresponding slip time t_s. One keeps Eq.3 but with $\rho(\boldsymbol{k},0) \to \rho(\boldsymbol{k},t_s)$, where t_s is, in analogy to the slip length, of the order of the collision time. The crucial result obtained by Kirkpatrick, Belitz, and Dorfman is that for systems with long-time tails, it is no longer possible to capture microscopic initial values through the concept of a slip time. In other words, even knowing the density after a several collision times have passed is not sufficient to predict its future using hydrodynamics alone. Instead, $\rho(\boldsymbol{k},t)$ is at large times inevitably affected by the initial values of non-hydrodynamic observables. The non-hydrodynamic initial values are not forgotten.

These results imply that there is a fundamental incompleteness of hydrodynamics. A consistent theory in terms the hydrodynamic modes alone seems not sufficient and higher order correlations affect the dynamics even at longest times. While these effects may be quantitatively small in many circumstances, the conclusion is nevertheless disconcerting. In the language of the three pillars of hydrodynamics listed above, it means that one cannot formulate a consistent description once the constitutive relations include fluctuation-induced non-analytic low-frequency behavior of the transport coefficients. It would be interesting to see whether these effects can be observed experimentally, most likely using noise spectroscopy.

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