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A pathway to parafermions

Induced superconductivity in the fractional quantum Hall edge
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Recommended with a Commentary by Jason Alicea, Caltech

The fractional quantum Hall effect and superconductivity are unnatural allies: the former thrives on strong magnetic fields whereas the latter generally abhors them. Negotiating their successful merger, however, promises transformative payoff. Namely, theory predicts that fractional quantum Hall/superconductor hybrids can host 'parafermionic' generalizations [1] of Majorana zero modes that have been intensely pursued over the last decade. The highlighted paper by Gül et al. establishes a milestone toward parafermions by unambiguously demonstrating induced superconductivity in fractional quantum Hall edge states. To put their results in proper context let us first address three questions: Why parafermions? How exactly can one engineer parafermions? And how might they be detected experimentally?

Majorana zero modes already comprise an enticing target for physicists seeking novel quantum phenomena with technological applications. In topological superconductor platforms, they represent novel zero-energy degrees of freedom that bind to defects such as domain walls or vortices, and are formally described by Hermitian operators γ_a satisfying

$$\gamma_a^2 = 1, \quad \gamma_a\gamma_b = -\gamma_b\gamma_a \text{ (for } a \neq b). \tag{1}$$

A well-separated pair of Majorana zero modes furnishes a single fermionic two-level system whose occupation can not be determined via local measurements (neglecting exponentially small effects). Arrays of Majorana modes accordingly generate a set of locally indistinguishable ground states that can encode qubits with built-in resilience against local environmental noise. Moreover, braiding the defects binding Majorana zero modes enacts rigid rotations within the ground-state manifold—i.e., they exhibit non-Abelian statistics—yielding intrinsically fault-tolerant quantum gates. What more could one ask for? Computational universality requires (for instance) a Hadamard gate, $\pi/8$ phase gate, and two-qubit entangling gate, and unfortunately Majorana braiding produces only the first of these [2]. Supplementing braiding with measurement additionally enables an entangling gate yet still falls short of universality. In search of fully fault-tolerant universal quantum computing hardware, we must look beyond Majorana platforms.

Parafermion zero modes are $\mathbb{Z}_N$ Majorana generalizations described by unitary operators α_i that obey

$$\alpha_a^N = 1, \quad \alpha_a\alpha_b = e^{i\frac{2\pi}{N}\mathrm{sgn}(a-b)}\alpha_b\alpha_a \text{ (for } a \neq b); \tag{2}$$

note that the $N = 2$ limit reduces to the Majorana properties from Eq. (1). Like their Majorana cousins, parafermion arrays span degenerate, fault-tolerant qubit states manipulatable by non-Abelian braiding operations. Parafermion braiding is richer, however, compared to the Majorana case and supplies an entangling gate *without* the need for measurement. While even there braiding remains computationally non-universal, two-dimensional parafermion arrays form natural building blocks for engineering so-called Fibonacci anyons, for which braiding *does* provide a universal gate set [3]. Long-term fault-tolerant quantum computing applications thus partially answer the 'Why parafermions?' question.

Further motivation derives from the inherent beauty in how parafermions emerge. Typically fractional quantum Hall states are viewed in terms of an Abelian/non-Abelian dichotomy depending on the braiding properties of the constituent anyons. For instance, Laughlin states belong to the former category because they host anyons with Abelian fractional statistics, whereas the Moore-Read state exemplifies the latter and features anyons displaying non-Abelian statistics. But quite stunningly, even the simplest Abelian fractional quantum Hall phases contain seeds of non-Abelian anyon physics that can be harvested via the introduction of certain kinds of defects.

Imagine carving a narrow trench into a Laughlin state, generating a set of counterpropagating fractionalized edge states within the material as depicted in Fig. 1(a). These modes can acquire a pairing gap in the presence of proximity-induced superconductivity. The result, however, is far from an ordinary superconductor. Cooper pairs descend from conglomerates of minimal e^* fractional charges supported by the Laughlin state, and *the induced pairing catalyzes condensation of charge-*$2e^*$ *composites*! The trench can thereby soak up fractional charges into the condensate without changing the system's energy—implying ground-state degeneracy. This degeneracy is captured by parafermion zero modes that reside at the left and right ends of such a one-dimensional fractionalized superconductor* [4, 5, 6, 7]; see Fig. 1(a). Physically, the parafermion operators add fractional charges to the trench endpoints, cycling the system through degenerate configurations allowed by the condensate.

Signatures of parafermion zero modes appear in electrical transport. In the geometry of Fig. 1(b), fractional quantum Hall edge states native to the outer sample boundary serve as leads that interrogate the parafermion mode on one side of the trench. At low energies the parafermion converts an incident negatively charged $-e^*$ fractional edge quasiparticle into an outgoing $+e^*$ fractional charge *with unit probability*, with the deficit $-2e^*$ charge absorbed by the nontrivial condensate. This remarkable phenomenon can be viewed as perfect (chiral) crossed Andreev reflection,† and manifests as a sign reversal of the electric potential for the outgoing edge states compared to the incoming edge-state potential.

Figure 2(a) shows the quantum Hall-superconductor hybrid device studied by Gül et al. The central rectangular area, composed of a graphene-based heterostructure, hosts quantum Hall states that are proximitized by a thin grounded NbN superconductor (blue region); note the similarity to the setup sketched in Fig. 1(b). The NbN superconductor benefits from both a high critical field and appreciable spin-orbit interaction, the latter of which is crucial for inducing superconductivity into spin-polarized quantum Hall states. Contacts along the

*Replacing the Laughlin state by a $\nu = 1$ integer quantum Hall phase yields Majorana modes instead of parafermions.

†Alternatively, the phenomenon is a fractionalized, chiral counterpart of (gasp!) the quantized zero bias conductance predicted for Majorana zero modes.

Figure 1: (a) Fractional quantum Hall-superconductor architecture hosting parafermion zero modes. (b) Perfect crossed Andreev reflection mediated by a parafermion zero mode: an incident $-e^*$ fractionally charged edge excitation converts, with unit probability, into an outgoing $+e^*$ fractional charge. The sign of the edge potential correspondingly flips for the outgoing edge mode.

periphery enable extraction of various voltages along the edge. Most interesting here are the potentials for the incoming and outgoing edge modes, respectively denoted V and $V_{\rm CAR}$ by the authors. These quantities are used to define a resistance $R_{\rm CAR} = V_{\rm CAR}/I$ (I denotes the injected current) along with a ratio $p_{\rm CAR} = -V_{\rm CAR}/V$. When incoming edge quasiparticles preferentially reverse their charge upon passing the superconductor, indicating a propensity for crossed Andreev reflection, $V_{\rm CAR}$ becomes negative and therefore so does $R_{\rm CAR}$. In this regime, $p_{\rm CAR}$ can be interpreted as a crossed Andreev reflection probability—which as noted above tends to unity at low energies in the presence of parafermion zero modes.

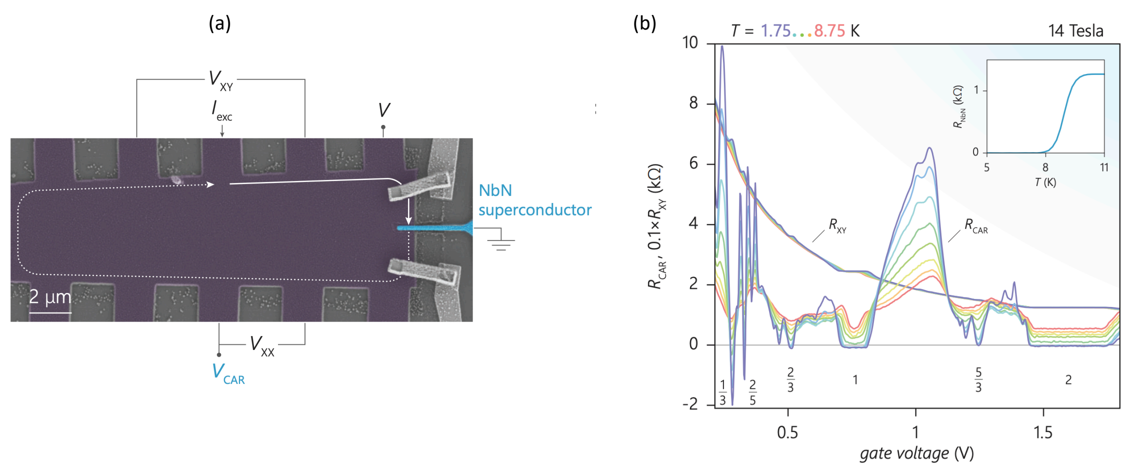

Figure 2: (a) Device studied by Gül et al. (b) Data illustrating the low-temperature onset of crossed Andreev reflection, revealed through negative $R_{\rm CAR}$ resistances, in both the integer and fractional quantum Hall regimes.

These measurements reveal a wealth of interesting, and surprising, results. Figure 2(b) illustrates Gül et al.'s main finding: as temperature is lowered, negative $R_{\rm CAR}$ resistances develop for a series of integer *and* fractional quantum Hall states—clearly evidencing induced superconductivity in both regimes. The corresponding p_{CAR} ratios remain much smaller than one in all cases but can approach respectable values of order 10%. Remarkably, for the integer quantum Hall states, p_{CAR} saturates at low temperatures to values that are nearly independent of filling factor ν from $\nu = 1$ all the way to $\nu = 6$. This observation is counterintuitive: One might have expected nontrivial filling-factor dependence based on the difference in spin polarization for even vs odd ν, combined with the nontrivial spatial distribution of integer quantum Hall edge states (e.g., edge modes located farther from the superconductor would naturally inherit a weaker superconducting proximity effect). Instead it appears as if the superconductor indiscriminately swallows up *all* incident integer quantum edge states and spits out negated charges on the other end with uniform probability. In the fractional quantum Hall regime the p_{CAR} probabilities are also perplexing. For $\nu = 2/3$, p_{CAR} saturates at low temperatures to a similar value as for the integer states, whereas for $\nu = 1/3$ and $2/5$, p_{CAR} continues to increase down to the lowest accessible temperatures.

While the relation of these results to parafermions is presently uncertain, a more general message is clear: Two classic problems—superconductivity and the fractional quantum Hall effect—have now been successfully integrated, opening up a fascinating frontier in strongly correlated electrons. The puzzles uncovered by Gül et al. provide welcome challenges for theory as the field progresses towards definitive experimental realizations of parafermions.

References

[1] Paul Fendley, J. Stat. Mech. (2012) P11020

[2] Das Sarma, S., Freedman, M., and Nayak, C., npj Quantum Inf **1**, 15001 (2015).

[3] Roger S. K. Mong, David J. Clarke, Jason Alicea, Netanel H. Lindner, Paul Fendley, Chetan Nayak, Yuval Oreg, Ady Stern, Erez Berg, Kirill Shtengel, and Matthew P. A. Fisher, Phys. Rev. X **4**, 011036 (2014)

[4] Netanel H. Lindner, Erez Berg, Gil Refael, and Ady Stern, Phys. Rev. X **2**, 041002 (2012)

[5] Clarke, D., Alicea, J., and Shtengel, K., Nat Commun **4**, 1348 (2013)

[6] Meng Cheng, Phys. Rev. B **86**, 195126 (2012)

[7] Abolhassan Vaezi, Phys. Rev. B **87**, 035132 (2013)