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Proposals to Realize Topological Superconductivity in Cuprates by Twisting and Stacking

1. **High-temperature topological superconductivity in twisted double layer copper oxides**
   Authors: Oguzhan Can, Tarun Tummuru, Ryan P. Day, Ilya Elfimov, Andrea Damascelli, Marcel Franz
   arXiv:2012.01412
   Nature Physics 17, pg. 519 (2021)
2. **Magic angles and current-induced topology in twisted nodal superconductors**
   Authors: Pavel A. Volkov, Justin H. Wilson, J. H. Pixley
   arXiv:2012.07860

Recommended with a Commentary by Ashvin Vishwanath, Harvard University

A long sought after goal has been to coax the nodal d-wave quasiparticles of the high temperature cuprate superconductors into a gapped, topological state. One may view the gapless nodal quasiparticles of the d-wave pairing ($d_{x^2-y^2}$ pairing) as being poised between two gapped phases - a trivial and a topological phase. A perturbation with the right properties could open the desired gap, resulting in a topological superconductor. The simplest variety of topological superconductor that can emerge in this setting is the so called $d_{x^2-y^2}+ i\, d_{xy}$ (or d+id for short) topological superconductor, which is characterized by chiral edge states which are one way propagating quasiparticle modes at the edge of every 2D sheet. An attractive feature that we may envision for such a topological superconductor is the potentially elevated temperature and energy scales given its realization in a high Tc superconductor.

The minimal symmetry requirement to obtain a d+id superconductor is that both time reversal symmetry and reflection symmetry (about any mirror plane that is perpendicular to the 2D sheet), must be broken. A vertical magnetic field satisfies these symmetry lowering requirements - indeed there was an active but inconclusive search for such d+id superconductivity in the mixed state of the cuprate superconductors about two decades back.

The featured references outline new blueprints for realizing topological d+id superconductivity in cuprates, stimulated by two recent advances.

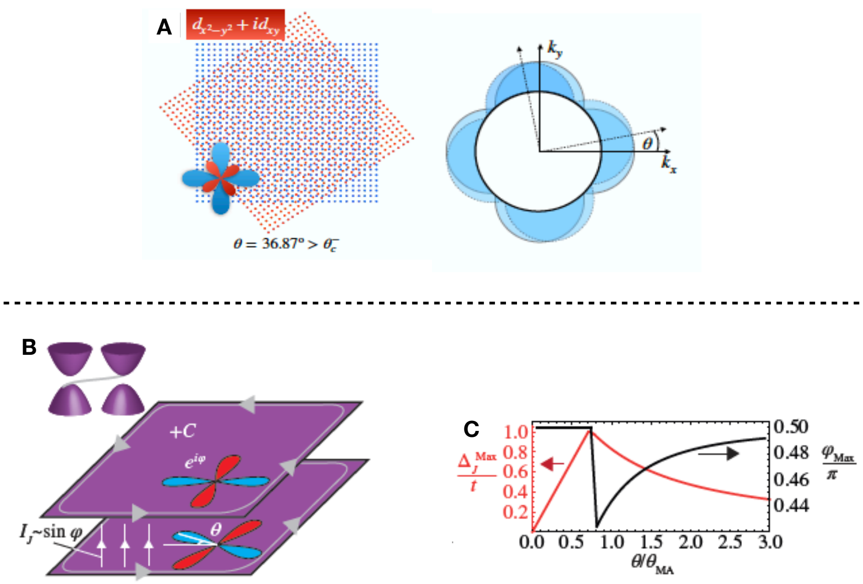

Figure 1: (A) From Reference 1, two d-wave superconductors twisted by a large angle. (B, C) From Reference 2, small relative twist angle with a vertical Josephson current, giving rise to a quasiparticle gap set by the interlayer tunneling strength.

The first advance is the rise of twistronics, following the explosion in interest in stacked and twisted 2D structures including multilayers of graphene and transition metal dichalcogenides. Stacking and twisting two layers of the cuprate superconductors suggests a natural variant of this problem, particularly in the superconducting state, where a low density of electronic excitations of the d-wave superconductor are expected. The second advance is the remarkable progress in isolating and measuring monolayer cuprates [1], which were demonstrated to retain their bulk behavior in the extreme 2D limit, including their high superconducting transition temperature .

Both references propose stacking a pair of d-wave superconductors and twisting them relative to one another by an angle θ. In **Reference 1**, this angle is large, $\theta \sim 45^o$. The logic goes as follows - Cooper pair tunneling between a pair of d-wave superconductor layers locks together their relative phases $\Delta\Phi = \phi_1 - \phi_2$ via the Landau free energy:

$$\mathcal{F} = -J(\theta)\cos\Delta\Phi + \mathcal{K}\cos 2\Delta\Phi$$

Typically we only focus on the first Cooper pair tunneling term, but due to the d-wave symmetry, the pair tunneling amplitude $J(\theta)$ changes sign i.e. $J(\theta) = -J(90^o - \theta)$ on reflecting about the diagonals. Thus, $J(\theta = 45^o) = 0$ since the Cooper pair tunneling is frustrated by the change in sign of the d-wave order parameter. At this point tunneling by *pairs* of Cooper pairs takes over, giving the second term (crucially with $\mathcal{K} > 0$). This term is minimized when the relative phase difference is $\Delta\Phi = \pm\pi/2$. Picking one of these phases corresponds to the breaking of time reversal and reflection symmetries. Furthermore, assuming this this broken symmetry state is realized, a gap opens at the Dirac nodes, resulting in a quantized topological invariant C. For a single layer of d+id superconductor this invariant is $C = \pm 2$. In the twisted bilayer, $|C| = 4$, 2, 0, depending on parameters such as the interlayer tunneling strength and Fermi surface dimension. A topological ground state is found that over a significant parameter range.

Reference 2 proposes a similar setup but at a small angle. Instead of appealing to spontaneous symmetry breaking, the requisite symmetry lowering is done by hand, by requiring the top and bottom layers to be at different phases, by passing a Josephson current in the vertical direction. Now, the combination of vertical current and the twist implies that the requisite symmetries are broken and the d-wave quasiparticles will once again be gapped. The gap can be optimized by tuning to a 'magic angle' where the velocity of the Dirac nodes vanish. This angle is estimated to be $\theta \sim 2t/v_\Delta K_D$, where t is the tunneling between layers, v_Δ is the superconductor gap induced Dirac velocity and K_D is the crystal momentum of the nodal point. Estimating t requires some care at small angles, since the dominant tunneling between copper oxide layers is expected to vanish near the nodes.

An obvious question for experiments is how these predictions can be tested. Probing the neutral chiral edge modes is challenging, especially since the probe of choice is thermal conductivity, which at low temperature T should reveal a quantized Hall signal: $\frac{\kappa_{xy}}{T} = C\frac{\pi^2 k_B^2}{6\hbar}$. Other more accessible probes include optical detection of the $d + id$ order parameter and tunneling signatures of the gapless edge modes. Looking further afield, one may wish to eventually realize a single Majorana chiral edge mode, in contrast to the $C = 4$ or $C = 2$ topological states where one has 8 and 4 chiral Majorana modes respectively. This will require breaking of spin rotation symmetry as well, perhaps by bringing the edge in proximity to a material with strong spin orbit coupling.

The current blueprints are based on simple models of superconductivity that do not incorporate magnetism and other correlation effects. A theoretical opportunity here is to examine if twisting can itself be used to probe, and perhaps even amplify, the effects of strong correlations in cuprates and other materials. Exploring these directions should help us scrutinize long studied quantum materials from a new angle.

References

[1] High-temperature superconductivity in monolayer Bi2Sr2CaCu2O8+. Yijun Yu, Liguo Ma, Peng Cai, Ruidan Zhong, Cun Ye, Jian Shen, G. D. Gu, Xian Hui Chen Yuanbo Zhang . Nature 575, pg 156 (2019).