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Hofstadter superconductors

Theory of Hofstadter Superconductors
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The ability to create two-dimensional materials with large unit cells in moiré structures opens up a whole new parameter regime to study effects of high magnetic fields; high in the sense that the flux per unit cell Φ becomes comparable to the flux quantum Φ_0. In this case one must of course properly account for the electronic structure forming a complex energy spectrum, the Hofstadter butterfly[1, 2, 3, 4]. In the high-field regime, superconductivity is full of surpises[5, 6]. While semiclassical reasoning suggests that superconductivity disappears at H_{c2}, when a flux quantum fits in the area ξ_0^2 given by the coherence length, it gets enhanced in the high-field regime, at least if one can ignore Pauli limiting effects[7]. This is a consequence of the altered density of states due to Landau-level quantization. From the theory of the quantum Hall effect it is well known that an additional periodic potential leads to an additional level complication as Landau levels split into q sub-bands if

$$\Phi = \frac{p}{q}\Phi_0. \tag{1}$$

Here, p and q are coprime.

In their manuscript Shaffer, Wang, and Santos give a very lucid and detailed analysis of the nature of the symmetry-allowed pairing states in periodic solids at high field as function of p and, more importantly, q. They find, in agreement with earlier results for charged bosonic systems[9, 10], that superconductivity inevitably implies a broken translation invariance. In addition, they show a rather peculiar aspect that is unique to the fact that Cooper pairs have an effective charge $e^\star = 2e$; different symmetry pattern occur for even and odd values of q. Finally, they analyze the topological properties of such Hofstadter superconductors and show under what conditions Bogoliubov Fermi surfaces[11] are to be expected.

Placing a two-dimensional electron system in a homogeneous magnetic field that points perpendicular to the plane does, by itself, not break translation invariance. However, gauge invariance implies that translations are tied to gauge transformations. This is ultimately the reason for the perfect Meissner effect at small and the Abrikosov vortex lattice at intermediate magnetic fields. Just like in the Abrikosov lattice, in the high-field limit the phase of the order parameter winds as one parallel transports around a unit cell. The combinations of $U(1)$ gauge transformations and translations form the magnetic translation group (MTG), consisting of non-commuting lattice translations. As MTG contains a subgroup of $U(1)$, broken $U(1)$ necessarily breaks at least some elements of the magnetic translation group. From their symmetry analysis Shaffer *et al.* demonstrate that due to $e^\star = 2e$ for Cooper pairs, the order parameter of the system has q components if q is odd and $q/2$ components if it is even. Under a subsequent action of translations $\hat{T}_1$ and $\hat{T}_2$ of the MTG along the two primitive lattice vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ of the kind $\hat{T}_2\hat{T}_1\hat{T}_2^{-1}\hat{T}_1^{-1}$ the order parameter picks up the Aharonov-Bohm flux of the unit cell

$$\hat{\Delta} \to e^{-i\frac{e^*}{e}\frac{2\pi p}{q}}\hat{\Delta}. \tag{2}$$

This ultimately leads to the different transformation behavior for odd and even q, where the parity of the total momentum of the pair leads to different transformation behavior for even q. This reasoning seems to naturally extends to more complex superconducting states such as those with $e^\star = 4e$ pairing, where one then expects different behavior, depending on the residue of q modulo 4. Quite beautifully this q- or $q/2$-component order parameter leads to a Ginzburg-Landau expansion with internal $\mathbb{Z}_q$ symmetry that can be analyzed using the established approaches of multi-component superconductors. These $\mathbb{Z}_q$-symmetric Hofstadter superconductors allow for topological superconductivity, where the Chern number can be shown to have the same parity as the integer q. Overall this is a wonderful example for the power of symmetry classifications in a complex and highly interesting setting.

The analysis by Shaffer *et al.* offers a very clear and comprehensive account of the mean-field behavior of Hofstadter superconductors. It is the starting point to address a number of interesting issues that one would expect or hope to be important in real materials: First, there is the role of Pauli limiting and spin behavior in general. One clearly expects this to affect the symmetry classification and maybe even the stability of the superconducting states in a substantial way. Second, fluctuations beyond mean field display very rich behavior for multi-component superconductors, where vestigial order seem inevitable. Hence, one expects partial symmetry breaking to take place already before the onset of superconductivity, leading to rich high-field phase diagrams. Third, given the crucial role of interactions at high magnetic fields, one would hope that this is the parameter regime where fractionalized anyon superconductivity might finally be realized.

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