Observation of KPZ super-diffusion in Heisenberg chain

[1] **Quantum gas microscopy of Kardar-Parisi-Zhang superdiffusion**
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Recommended with a Commentary by Tin-Lun Ho, The Ohio State University

In the last decade, there have been increasing number of studies of spin diffusion in solid state and cold atom systems. These studies explore the effect of strong interactions on spin excitations. At the same time, they address the fundamental issue of thermalization, as it is easy to create experimentally highly non-equilibrium spin states by rapidly changing the magnetic field, and to observe their relaxations. From general arguments[2], one expects the motion of spin to be diffusive, described by some type of hydrodynamics. The diffusive behavior, characterized by a temporal power law $t^{1/z}$, reflects the nature of spin excitations. The exponent z will depend on whether the excitations behave like Brownian particles, ballistic particles, or else. The hydrodynamics governing the spin diffusive is the result of the thermalization of the initial non-equilibrium state. How quantum systems that evolve unitarily achieve hydrodynamic behavior is a fundamental question being actively studied today.

Theoretical studies have focused mostly on 1D quantum spin chains, as they are better understood than their higher dimensional counterparts. In addition, 1D quantum spin chains are integrable, which lead to large number of conserved quantities. Tuning the system away from integrability will also illustrate how additional conserved laws affect thermalization. Despite the exact Bethe Ansatz solutions, the studies of the dynamics 1D spin chains remain formidable, as there are no simple ways to apply the Bethe Ansatz results to far from non-equilibrium processes. Most current studies are based on numerical simulations. Recently, it is found that for the Heisenberg model has an exponent $z = 3/2$. (See ref [13-19] in ref.[1]). The behavior is characterized as "superdiffusion" as it is faster than the Brownian diffusion $z = 2$. The behavior $t^{2/3}$ is also the result of the celebrated Kardar-Parisi-Zhang (KPZ) equation. The KPZ equation is a non-linear stochastic partial differential equation originally designed to describe the dynamics of the interface in crystal growth. Later, it is found that it applies to a great variety of systems. It has now become a major area of research in Mathematics. The discovery of KPZ dynamics in Heisenberg chain is a surprise. First of all, evolution of spin is unitary, while the KPZ equation contains a stochastic external force.

Secondly, the Heisenberg model has SU(2) symmetry, while the KPZ equation describes a classical scalar field. Despite clear numerical evidence, how KPZ dynamics emerges from the quantum dynamics of the Heisenberg chain remains an open question.

The recent experiment the Munich group [1] has addressed many of the questions above and more. The Munich group has engineered a spin-1/2 XXZ chain with 50 sites and study their dynamics. These studies are made possible by several major advances in cold atom experiments: the ability to control the parameters of an optical lattice, the ability to address specific spin states of an atom that is necessary for initial state preparation, and the ability to detect the locations of atoms with single site resolution. To create an XXZ chain, the Munich group creates a Mott phase of a Rb Bose gas with one boson per site in a 1D tight-binding lattice. The Rb bosons are in one of two hyperfine states (F=1 and F=2). They can be regarded as pseudo-spin bosons. Due to the double exchange and the overlap of the wavefunctions of neighboring bosons, the effective hamiltonian is that of a spin-1/2 XXZ chain, $H = -J\sum_i(S^x_iS^x_{i+1} + S^y_iS^y_{i+1} + \Delta S^z_iS^z_{i+1})$. For Rb atoms, Δ is about 1. So the spin chain is close to the Heisenberg limit. The total magnetization of the system can be adjusted by varying the number of bosons in the F=1 and F=2 states. In addition, the system can also be tuned away from integrability by coupling a 2D array of chains.

To study spin diffusion, the Munich group prepares an initial state with a domain wall where the magnetization on the left and the right half of the chain is $\delta + \eta$ and $\delta - \eta$. That is to say, the total polarization of the entire chain is δ, and the difference of polarization between left and right part of the chain is 2η. This preparation is achieved by addressing specific spin states of the Rb atom. Spin diffusion is studied by tracking the polarization transfer, $P(t)$, defined as the total number of spins which have crossed the domain wall by time t. The tracking is performed using the powerful atom microscope that detects the location of an atom with single site resolution. The long time behavior of $P(t) \sim t^(1/z)$ was measured for different magnetizations and for different couplings between parallel chains. The result is shown in a figure in ref [1], which we reproduce below. KPZ hydrodynamics $z = 3/2$ (or $P(t) \sim t^{2/3}$) is indeed found, verifying the previous numerical results. As the coupling between chains increases, the system loses integrability. The diffusion reduces to Brownian diffusion with $z = 2$ (or $P(t) \sim t^{1/2}$) as implied by standard spin hydrodynamics. On the other hand, $z \sim 1$ (or $P(t) \sim t$) as magnetization increases, reflecting the ballistic motion of a dilute gas of spin down in a “vacuum” made up of spin up bosons. The speed of spin diffusion of the Heisenberg model is in between the ballistic and the Brownian diffusion. The authors in ref.[1] also obtain the KPZ scaling function by collapsing different spatial-temporal profiles.

The Munich experiment is yet another demonstration of the power of quantum simulation. The confirmation of the previous numerical predictions with precision means that these methods can be applied to cases where numerical results are not available, such as in the higher dimensions. The advances in quantum gas microscopy has played a key role in the current development of Quantum Simulation. The fact that one can take a snapshot of the many-body wavefunction, obtain averages of observables through the prescriptions of Copenhagen Interpretation of Quantum Mechanics, and extract important information from noise statistics is an extraordinary achievement in quantum measurement. These imaging techniques complement well the transport measurements in solid state systems, and is a new way to explore many body phenomena. One can be sure a lot more information of fundamental interest will be revealed by quantum gas microscopy in the future.

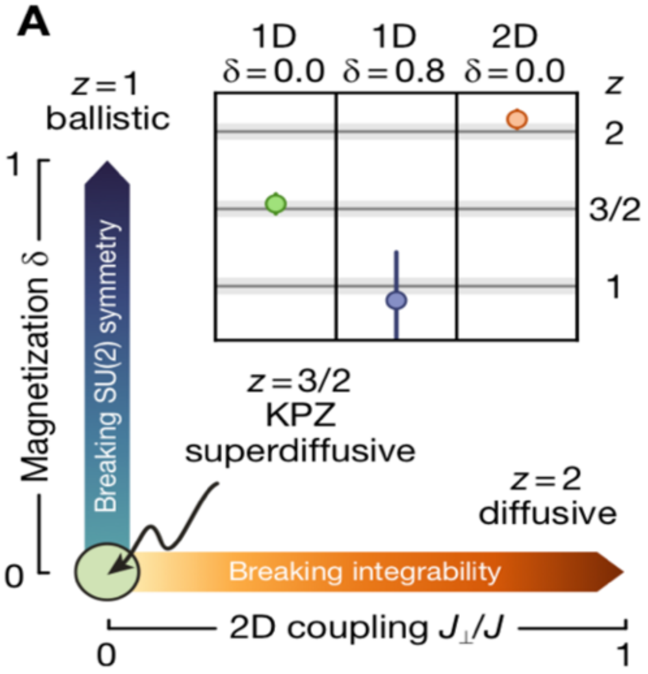

Figure 1: The value of the exponent z as a function of magnetization and coupling between chains shown in Ref.[1].

References

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[2] Leo Kadanoff and Paul Martin, Hydrodynamic Equations and Correlation Functions, Annals of Physics 281, 800-852 (2000)