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Flying with q directions: Consequences on flocking transition

Susceptibility of Polar Flocks to Spatial Anisotropy
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While active matter in general is the subject of extremely active current research, the role of external spatial anisotropy is still largely unexplored. Yet, such an anisotropy is effectively present in most experimental conditions where active systems are investigated: for instance, the presence of weak external fields typically biases the dynamics of particles' orientation along some directions; otherwise, geometrical constraints of specific systems can explicitly break rotational symmetry at the microscopic level. In both cases, the system is not invariant under infinitesimal rotation of all the particles' orientation vectors, *i.e.* $\theta_i \to \theta_i + \mathrm{d}\theta$.

Here we comment on the paper by Solon *et al.* in which the authors assess, in a quantitative way, the role of anisotropy on the collective behavior in a class of *active clock models* in two spatial dimensions. These models describe the dynamics of self-propelled particles diffusing on a lattice. The probability to jump between sites is biased by the direction of an individual orientation vector that can only take q discrete values. The orientation of each particle is subject to local alignment with its neighbors. The cases $q = 2$ (active Ising model, AIM [1]) and $q \to \infty$ (Vicsek model, VM [2]) are two paradigmatic instances of aligning active matter, which exhibit a flocking transition with coexistence between an apolar disordered state and a polar state with long-ranged order.

Importantly, although the flocking transition emerges through a phase separation in both AIM and VM, yielding a coexistence regime where polar bands of aligned units travel in an apolar background, there are still important qualitative differences between these models. In AIM, the system phase separates into a single, dense polar band surrounded by a gas-like, apolar phase (macrophase separation); in the fully ordered state, the correlations are short-ranged, no giant density fluctuations appear, and the global orientation is pinned throughout the dynamics [3]. In contrast, for VM, phase separation occurs with many traveling polar bands separated by a dilute apolar gas (microphase separation); in the fully ordered state, correlations are scale-free, giant density fluctuations arise, and there is a wandering global orientation [4].

The main mechanism underlying these differents scenarios, namely either macrophase or microphase separation, is the existence (breakdown) of continuous rotational invariance for VM (AIM). For equilibrium models of order-disorder transitions in two dimensions, it is well known that, whenever $q > 4$, there is a long-range (LR) ordered phase below a critical temperature T_q, followed by a quasi-long-range (QLR) ordered phase at $T_q < T < T_{\mathrm{BKT}}$ [5]. In short, anisotropy matters in equilibrium since, even in the thermodynamic limit of large system size L, one can induce a transition from QLR order to LR order by tuning temperature (Fig. 1). Besides, note that the critical temperature T_q vanishes for $q \to \infty$, owing to the celebrated Mermin-Wagner theorem for equilibrium systems [6], which precludes any LR order for the XY model in 2D.

Active clock models evade equilibrium constraints, enabling LR order at finite temperature even for systems with continuous symmetries, such as in VM. The question addressed by the authors is then whether, in the thermodynamic limit, anisotropy for any finite q larger than 2 leads the system to have the phenomenology of either AIM or VM. For instance, in the coexistence region, can one induce a transition from microphase to macrophase separation by changing q? The answer provided by the authors in [7] is that, at large L, any active clock model at finite q falls into the class of AIM, namely $q = 2$. This behavior is made evident by quantifying the transverse magnetization structure factor in numerical simulations, which allows to extrapolate a crossover length ξ_q: for system sizes $L \gg \xi_q$, the system exhibits AIM behavior, whereas it exhibits VM behavior for $L \ll \xi_q$. The origin of ξ_q is then rationalized through two approaches:

- A mean-field hydrodynamic theory which amounts to introducing an effective particle-based potential V_q, with explicit dependence on anisotropy q, acting on individual orientation. This potential accounts for the energy barriers created by anisotropy at finite q, which impede the rotation of the orientation vector. Interestingly, this microscopic potential damps the perturbations of the homogeneous ordered state at hydrodynamic level, with a damping length comparable with the crossover length ξ_q.
- A scaling argument to assess the relevance of V_q in the thermodynamic limit of large L. The authors extract a critical length scale L_c above which the anisotropy of microscopic alignment cannot be neglected at macroscopic level. They show that $L_c \sim \exp[q^2\sigma^2/(2z)]$, where σ^2 is the variance of polarization order around the homogeneous ordered state, and $z \simeq 1.33$ a dynamic exponent. Measuring numerically the variance σ^2 leads to values of L_c in good agreement with the crossover length ξ_q.

Overall, the paper describes how two different phenomenologies can actually arise from the same dynamics depending on system size. While the ultimate fate of any anisotropic, finite-q system is the same as that of AIM in the thermodynamic limit of large L, the paper quantitatively predicts the crossover length below which anisotropy becomes irrelevant. Interestingly, such finite-size effects are indeed to be taken into account in most experimental realizations of active matter, where the number of units is typically far from the thermodynamic limit, with drastic consequences on the emerging phenomenology. Two main research directions are left open by the authors: i/ a physical interpretation, based on microscopic mechanisms, to rationalize the emergence of either macrophase or microphase separation scenarios, as observed respectively in AIM and VM model, and ii/ an analytical prediction of polarization fluctuations σ in terms of microscopic control parameters, such as density and strength of alignment.

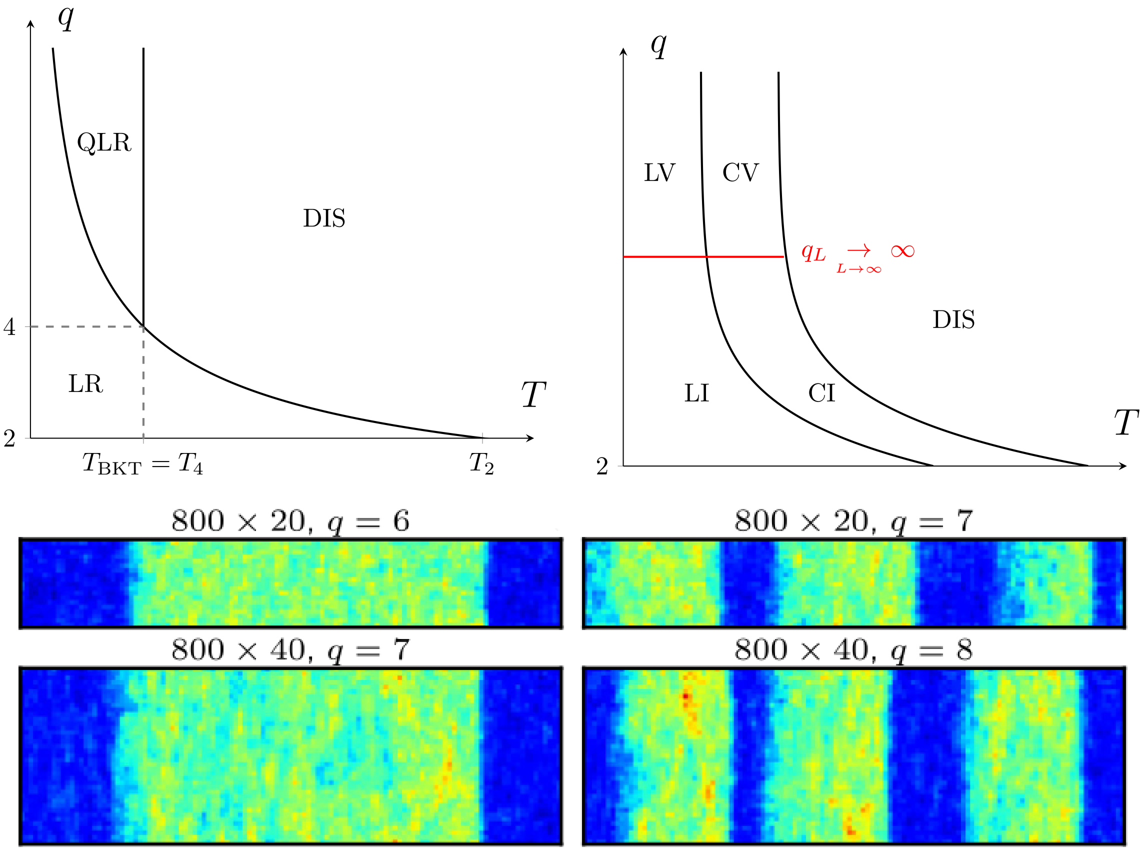

Figure 1: (Top left) Phase diagram in terms of anisotropy parameter q and temperature T for the equilibrium clock model in the thermodynamic limit of large system size L, following the scaling argument in [7]. When $q \leq 4$, there is only an Ising-like transition from LR order to disorder increasing T. When $q > 4$, there is LR order for $T < T_q$, QLR order for $T_q < T < T_{\mathrm{BKT}}$, and disorder for $T > T_{\mathrm{BKT}}$. When $q \to \infty$, namely for XY model, there is only QLR order since $T_q = 0$. (Top right) Sketch of the (q, T) phase diagram *at finite* L for the active clock model, where T is proportional to the inverse of aligning strength; adapted from Fig. 3(c) in [7]. The polar liquid phase has either VM behavior (LV) or AIM behavior (LI). Besides, the coexistence region shows either microphase (CV) or microphase (CI) separation. Increasing q, it is always possible to find VM behavior at finite L, but the crossover value q_L diverges with L. Hence, in the thermodynamic limit, any active clock model is in the same class as AIM ($q = 2$). (Bottom) Snapshots of density fields showing crossover from macrophase to microphase separation when increasing q at fixed L and T; light green corresponds to travelling polar bands at high density, blue indicates dilute apolar regions. Adapted from Fig. 3(b) in [7].

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