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Why aren't there any (normal) liquids much less viscous than water?

Minimal quantum viscosity from fundamental physical constants
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I particularly recommend the author's own brief Physics Today summary [1] of their work. They start this summary, appropriately, by citing the famous article by E. M. Purcell on swimming at low Reynolds numbers [2], where in the introduction Purcell states: "... if you look at the Chemical Rubber Handbook table you will find that there is almost no liquid with viscosity much lower than that of water. The viscosities have a big range *but they stop at the same place*. I don't understand that."

Authors of the recommended work offer a possible answer to this Purcell's puzzle which I find fascinating and thought provoking. In the end of the day, their claim is that kinematic viscosity of any normal liquid must satisfy

$$\nu \gtrsim \frac{1}{4\pi}\frac{\hbar}{\sqrt{m m_e}} \ , \tag{1}$$

where m_e and m are electron and atom masses, respectively.

One way to start making sense of this formula is to think of a diffusion of one atom of mass m in a liquid. If we imagine diffusion as a series of random steps of the size $\pm\Delta x$ performed with velocity $\pm\Delta v$, then Δx and Δv are constrained by the uncertainty principle, $\Delta x \cdot m\Delta v \gtrsim \hbar$, which means diffusion coefficient is bound from below by $D \sim \Delta x \cdot \Delta v \gtrsim \hbar/m$.

Authors offer several explanations as to why kinematic viscosity, which of course describes diffusion of momentum, has its lower bound by a factor of $\sqrt{m/m_e}$ larger. One easy argument begins with the Maxwell relation $\eta = G\tau$, where η is (dynamic) viscosity, G is modulus, and τ is relaxation time. Bulk modulus is estimated as $G \sim \varepsilon/a^3$, where energy is of order of Rydberg, $\varepsilon \sim m_e e^4/\hbar^2$ (e is of course the electron charge, and I use Gauss units), while distance is of the order of the atom size or Bohr radius, $a \sim \hbar^2/m_e e^2$. Finally, relaxation time τ can be estimated, for instance, by the virial theorem $\varepsilon \sim m(a/\tau)^2$; $1/\tau$ can also be understood as Debye frequency. Finally, kinematic viscosity $\nu = \eta/\rho$, where density is $\rho \sim m/a^3$. Collecting all of this together yields formula (1) – modulo, of course, the $1/4\pi$ coefficient.

I must admit that the origin of the coefficient $1/4\pi$ is still unclear to me, although I see that it helps when the numbers are plugged in and the result is compared to experimental data, as the authors had done. The coefficient is unclear, because none of the arguments presented by the authors appears to be much more than a scaling estimate. Nevertheless, the argument is interesting.

An earlier work [3], quite surprisingly, connects the question of minimal viscosity to the physics of black holes.* While it is widely known that black holes have entropy and temperature, authors consider the so called black branes, which are black holes with the translational invariant horizons, and show that long wave lengths perturbations in such objects can be described by hydrodynamics. In particular, hydrodynamic behavior of the horizon is identified with viscosity in the dual quantum field theory. These authors came up with the statement that

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \; , \tag{2}$$

where η is dynamic viscosity, s is volume density of entropy (and I use energy units for temperature, such that Boltzmann constant is equal to one, and entropy is dimensionless). In this case, coefficient 4π is indeed derived. Unfortunately, estimate (2), if one bravely applies it to regular liquids, produces numbers almost two orders of magnitude too small. Trachenko and Brazhkin argue that their estimate (1) is better, because it takes account of the inevitable for any condensed matter short distance cutoff at atomic scale.

In my opinion, the simplicity of the suggested formula (1), compared to the obvious complexity of the question, is nothing short of charming. Nevertheless, one obvious question remains open: why does water, of all liquids, nearly saturate the fundamental quantum physics lower bound of viscosity? Are all other puzzling properties of water related to this? Including the fact that water appears to play such a fundamental role for life?

References

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*I would like to thank Chandra Varma for pointing me to this article.