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Entanglement measure for Kondo spin at finite temperatures

Universal Thermal Entanglement of Multichannel Kondo Effects
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Recommended with a Commentary by Masaki Oshikawa, Institute for Solid State Physics, University of Tokyo

Kondo effect is one of the most important phenomena in condensed matter physics [1]. It may be regarded as the simplest system in which electron correlations are essential. The correlation effect can be systematically handled by Renormalization Group framework. In fact, Kondo effect is also one of the earliest examples of successful applications of the Renormalization Group approach [2].

While the "correlation" has been a very important concept, it has been discussed in condensed matter physics often without a precise quantification. Recent developments led to renewed understanding of quantum many-body physics. In particular, the concept of "quantum entanglement" has been imported from quantum information theory to condensed matter systems [3]. Very roughly speaking, quantum entanglement is a refined version of (quantum) correlation, with various entanglement measures [4] which are mathematically well defined. On the other hand, it is generally challenging to probe those entanglement measures in condensed matter experiments, although there have been numerous works in this direction. Perhaps the most popular entanglement measure in condensed matter physics literature is "entanglement entropy." However, it ceases to be a good entanglement measure when the entire system is in a mixed state, in particular at finite temperatures. Entanglement negativity is one of the standard entanglement measures which can also be used at finite temperatures. It has been calculated in various quantum many-body systems resulting in numerous interesting findings [5, 6]. However, in general it appears quite challenging to measure the entanglement negativity experimentally in condensed matter systems.

In the highlighted paper, the entanglement negativity between the impurity spin with $S = 1/2$ and the rest of the system in the Kondo problem was discussed. The temperature dependence was predicted based on conformal field theory (CFT), which was then verified with numerical renormalization group (NRG) calculations.

Since the Kondo problem can be formulated as an impurity in 1 dimensional free electron system, it is also a natural playground for conformal field theory (CFT) techniques. The RG fixed points of the problem naturally correspond to conformally invariant boundary conditions. This approach is particularly useful for overscreening multichannel Kondo problem [7]. In such a case, the infrared RG fixed point which governs the low-energy limit of the problem corresponds to neither the decoupled impurity spin nor to the complete screening of the impurity spin. It rather represents a nontrivial "local Non-Fermi Liquid" in which the correlation is essential. This is quite remarkable, considering the fact the interaction is present only at the impurity site, at least in the standard model for the Kondo problem. This is an example of the general feature of CFT that the conformally invariant boundary conditions can be nontrivial due to the interactions at boundary, even when the interaction is absent in the bulk [8]. The nontrivial conformally invariant boundary conditions of CFTs corresponding to the infrared RG fixed points for the multichannel Kondo problem have been constructed exactly.

The entanglement of the impurity spin with the bulk degrees of freedom was studied earlier [9]. The entanglement entropy is related to the universal boundary entropy ("ground-state degeneracy") which plays an important role in boundary CFT [10]. Despite these nice observations, there are several obstacles for a potential experimental observation of the entanglement. First, as discussed earlier in this commentary, the entanglement entropy is a good entanglement measure only at zero temperature. It is much desirable to discuss finite temperatures for potential experimental observations. Second, the entanglement entropy discussed in Ref. [9] actually does not measure the entanglement between the impurity spin and the rest of the system. This is because a field theory description generally relies on a coarse graining. The conformally invariant boundary condition actually describes not only the impurity spin but also electron degrees of freedom around the impurity. While this is quite natural for the field theory description and there is nothing wrong with this, it poses an additional challenge for potential experimental measurements. In fact, numerical verification of the theory was already somewhat complicated by this, although it was still manageable.

In the highlighted paper, the authors discussed the entanglement negativity, which is a valid entanglement measure even at finite temperatures, between the impurity spin and the rest of the system. When the Kondo impurity has $S = 1/2$, the Hilbert space of the impurity spin is only 2-dimensional. Because of this, the entanglement negativity between the impurity spin can be explicitly analyzed. Unlike in the earlier work [9], the one party of the entanglement is just the impurity spin and does not involve electron degrees of freedom near the impurity. Despite this, the entanglement negativity is related to boundary operators of the CFT describing the problem. The temperature dependence of the entanglement negativity exhibits power-law scaling according to this CFT picture. The CFT predictions indeed agree very well with numerical results obtained with NRG [11], also in the multichannel case which corresponds to local Non-Fermi Liquids.

Besides the interests in the new formulation and results themselves, this work seems to suggest new directions in studying quantum entanglement in condensed matter physics. First, by considering the entanglement between the impurity spin with a small and finite dimensional Hilbert space and the rest of the quantum many-body system, it introduces an intermediate step between the entanglement in small finite systems and the entanglement between large subsystems. This already simplifies numerical simulations and perhaps also experimental ones. Moreover, based on the relation between the entanglement negativity to local operators in field theory, it is suggested that the entanglement negativity may be estimated from response of the system to external perturbations such as the gate voltage. Measurements of such responses are rather standard in condensed matter physics experiments. We might hope that, in the future, entanglement will become a popular concept in

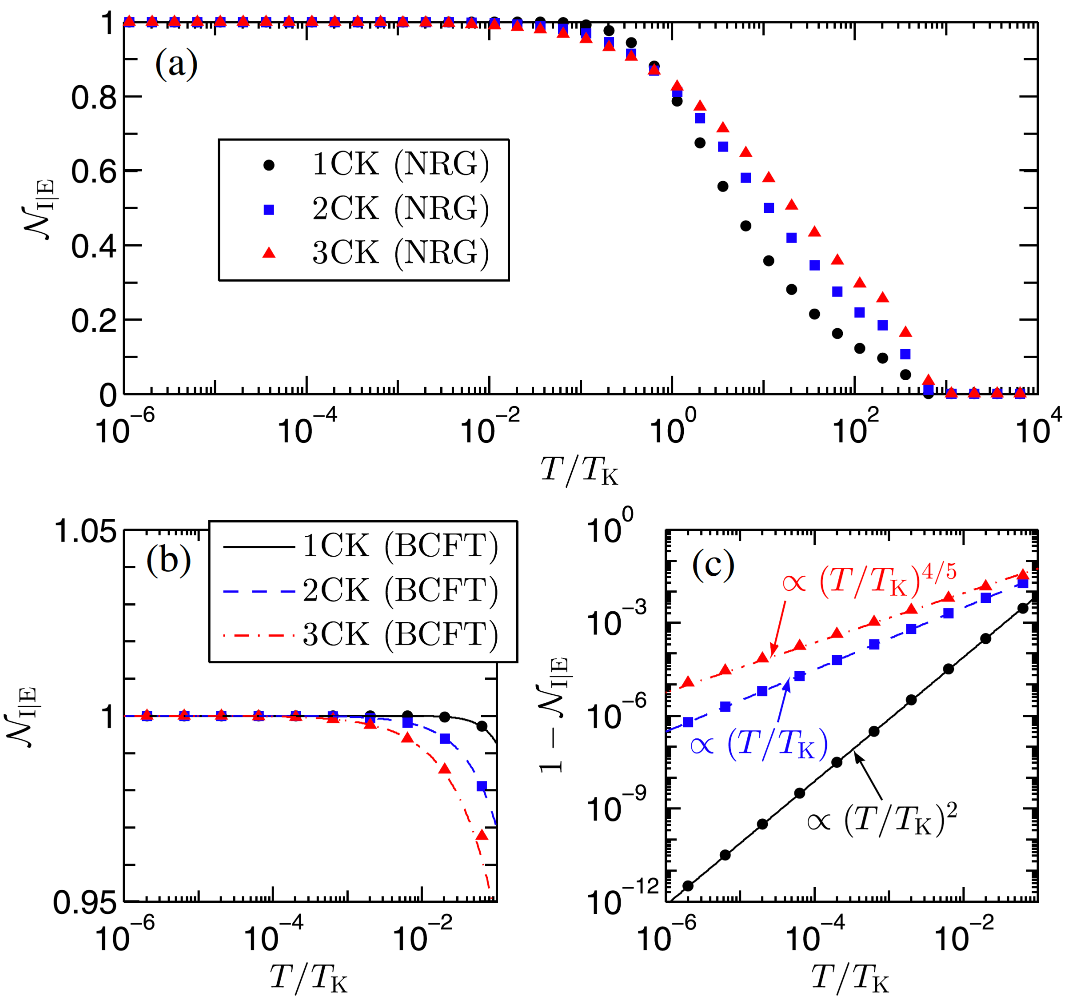

Figure 1: The entanglement negativity $\mathcal{N}_{I|E}$ between the impurity spin with $S = 1/2$ and the electrons obtained with numerical renormalization group method, as a function of the temperature in the k-channel Kondo problem ($k = 1, 2, 3$). The deviation from the maximum value 1 exhibits power-laws, in agreement with the conformal field theory prediction. (taken from the highlighted paper; Copyright American Physical Society (2021))

experimental condensed matter physics, similarly to topology these days.

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