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Conspiracies of clicks can change critical correlations

Measurements conspire nonlocally to restructure critical quantum states
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Recommended with a Commentary by S.A. Parameswaran, Rudolf Peierls Centre for Theoretical Physics, University of Oxford

Writing in this Journal Club a little more than a year ago, I flagged a flurry of activity exploring a new set of questions related to "hybrid" quantum dynamics in models where unitary evolution is punctuated by measurements. As sketched in my JCCM article, the early excitement centred on a phase transition where the generation of entanglement by the unitary evolution is sufficiently inhibited by measurements to trigger a qualitative change in the late-time state of the system. Many open questions remained, but diverse aspects of the problem have since been explored in several works. For the most part, however, the focus has been on models of qubits on a lattice with local gates, designed to resemble quantum computing devices, rather than the solid-state or ultracold atomic gas settings familiar to most condensed-matter physicists. A further challenge is that — with a few exceptions [1] — nearly all the proposals for interesting phenomena rely on post-selecting on the outcomes of measurements, which while conceptually reasonable is exponentially prohibitive in practice.

The recommended paper by Garratt *et al* partially addresses both of these issues. They consider a class of measurement-driven transitions in a Tomonaga-Luttinger liquid, which is the low-energy effective description of a wide variety of one-dimensional systems. They also propose creative ways to evade postselection. Together, these gently nudge the study of 'measurement-induced' phenomena towards a richer variety of experimental settings than proposed to date.

The basic problem posed by Garratt *et al* is to start with the ground state $|\psi_{\text{g.s.}}\rangle$ of some local Hamiltonian H, and subject it to a spatially extensive set of weak local measurements. Such measurements modify the amplitudes of different contributions to the many body state, but crucially do not fully disentangle the different system degrees of freedom, so that the final state after measurement is still nontrivially entangled. The question they ask is: to what extent do the measurements modify asymptotic properties of the state, such as the long-distance behaviour of equal-time correlation functions? For this to have maximally interesting answers, one should pick $|\psi_{\text{g.s.}}\rangle$ that have long-range entanglement, since this will be the most sensitive to the the influence of weak measurements. That said, too much entanglement is also not good: for example thermal states are highly entangled, but measuring one observable provides almost no information about other observables, and hence cannot modify expectation values far away. The ideal choice is thus a ground state with algebraic correlations between observables, since these typically have entanglement scaling intermediate between the area law of gapped ground states and the volume law exhibited thermal states. Such states usually appear at quantum critical points or in gapless phases of matter, which arise in a variety of settings in condensed matter physics.

Accordingly, the authors' choice of H is a simple model that supports critical entanglement without fine-tuning: namely, a single-channel gapless Tomonaga-Luttinger (TL) liquid. This emerges (for example) as the bosonized description of a spinless fermion chain [2]. Given the notorious differences in bosonization convention between different authors, it is perhaps worth writing down some details to fix notation for the rest of the commentary. At long wavelengths, the imaginary-time action, appearing for instance in the path-integral representation of the partition function $\mathrm{Tr}e^{-\beta H}$, is that of a free compact* bosonic field

$$S[\phi] = \frac{1}{2\pi K}\int dx \int_0^\beta d\tau \, \left[(\partial_\tau\phi)^2 + (\partial_x\phi)^2\right] \tag{1}$$

where $\beta = 1/k_BT$ is the inverse temperature. The field ϕ is related to the electron density via $n(x) = -\pi^{-1}\partial_x\phi + \pi^{-1}\cos[2(k_Fx - \phi(x))]$ in units where the lattice spacing is unity, k_F is the Fermi velocity, and we higher oscillations of the density beyond those at $2k_F$ have been elided. As is well-known, the correlations in the chain are governed by the 'Luttinger parameter' K, with $K = 1$ corresponding to the case of non-interacting fermions. While the system remains critical (and hence gapless) for all values of K in the absence of a lattice potential, the dominant correlation functions depend on K: equal-time correlations of the density and the phase θ (canonically conjugate to ϕ) in the TL ground state are given by

$$\langle n(x)n(0)\rangle \sim \frac{A}{x^2} + \frac{B\cos(2k_Fx)}{|x|^{2K}}, \qquad \langle e^{i\theta(x)}e^{-i\theta(0)}\rangle \sim \frac{1}{|x|^{1/2K}} \tag{2}$$

where A, B are real numbers, and the two terms in the first equation capture the smooth ($q = 0$) and $q = 2k_F$ 'charge density wave' (CDW) components of the density. Physically, $K > 1$ for attractive interactions and $K < 1$ for repulsive ones; this is reflected in the dependence of the correlations on K. To wit, smaller values of K correspond to slower decay of CDW correlations; larger values of K correspond to a slower decay of phase (superconducting) correlations. Note that when the charge decays more slowly, the phase decays more quickly, and vice versa, consistent with the fact that they are conjugate variables. (Of course, absent a lattice no true long-range order can be established due to the Mermin-Wagner theorem.)

Weak measurements are introduced as follows. Consider measuring the density locally at a point x_0, e.g. by shining light on the sample and measuring the scattered photon with a detector that has two possible states, $|0\rangle$ and $|1\rangle$. If no particle is present at x_0 then the detector remains in its initial state $|0\rangle$, but if a particle *is* present at x_0, it either remains in $|0\rangle$ or else 'clicks' i.e. changes its state to $|1\rangle$. Thus the 'no click' outcome leaves the density uncertain, thereby only weakly suppressing the particle density at x_0. This is in contrast to a perfect detector, where 'no click' would *definitely* correspond to the absence of a particle.

Garratt *et al* build a field-theoretic picture of the post-measurement state as follows: they first write the ground-state density matrix $|\psi_{\text{g.s.}}\rangle\langle\psi_{\text{g.s.}}|$ as the $\beta \to \infty$ limit of the imaginary-time evolution under $e^{-\beta H}$, with ground-state correlations then corresponding to equal-time

*This simply means that ϕ and $\phi + 2\pi$ label equivalent configurations.

correlators in this problem, which is controlled by the action $S[\phi]$. Using the standard technology of quantum measurements, for the set of outcomes m, the post-measurement state is $|\psi_m\rangle = M_m|\psi_{\text{g.s.}}\rangle/\sqrt{p_m}$, where M_m is a non-unitary operator encoding the weak measurement with outcome m, which occurs with Born probability $p_m = \langle\psi_{\text{g.s.}}|M_m^2|\psi_{\text{g.s.}}\rangle$. Putting these ideas together, in the TL case correlations in the state that results after measurement outcome m can be computed via a path integral involving the action (1). Since the choice of measurement commutes with both the field ϕ and the observable of interest (the density), and that acts throughout space, the measurements appear as perturbations of the one-dimensional surface at $\tau = 0$ in the two (spacetime) dimensional path integral. This translates the question about the longwavelength correlations in the post-measurement state to one of whether the perturbations representing M_m alter the correlations at $\tau = 0$, which in turn can be couched as question of relevance or irrelevance within a renormalization-group approach to the TL ground state.

The challenge confronting the authors of the recommended paper is how to extract non-trivial consequences of measurement. Naively averaging observables over measurement outcomes will not do: this is because averaging quantities *linear* in the density matrix with the Born probabilities p_m simply produces dephasing in the basis of measurement operators (this is essentially what is captured by the Lindblad dynamics of open quantum systems [3]). In order to average quantities nonlinear in the density matrix, such as squared correlators, they employ the 'replica method', a standard technique for taking formal powers of complicated objects (such as partition functions) that has been often invoked in studies of hybrid dynamics. Since the technicalities of the replica field theory calculations are lucidly explained by the authors, I focus the rest of this commentary on their results.

The first result considers a *single* very special 'no click' state $|\psi_{\text{n.c.}}\rangle$ by post-selecting the outcome in which none of the detectors register the presence of a particle with certainty. The field-theoretic formulation of this problem leads to an beautiful conclusion: the problem of understanding a quantum state perturbed by measurements at all all locations in space x but at a single time $\tau = 0$, can be mapped to an 'impurity problem' where the system is perturbed by a Hamiltonian that acts in a spatially local faction at $x = 0$ but at all times τ. In the path integral language one can view this as a Wick rotation that exchanges space and (imaginary) time, enabled by the Lorentz invariance of long-wavelength physics of the Luttinger liquid. Impurity problems have a storied history dating to studies of the Kondo problem, but the most salient result is the seminal analysis of a single scatterer in a Luttinger liquid by Kane and Fisher (KF, [4]). Translating KF's renormalization group analysis to the measurement problem, Garratt *et al* argue that, if the Luttinger parameter $K > 1$, the measurements are RG irrelevant and $|\psi_{\text{n.c.}}\rangle$ are $|\psi_{\text{g.s.}}\rangle$ have identical, algebraic correlations. In contrast if $K < 1$, then even for an arbitrarily weak measurement the algebraic decay of the correlations are modified: intuitively, since 'no click' measurement weakly suppresses density *fluctuations* and hence also the *correlations* of the conjugate phase variable (Intuitively, fluctuations of the phase are enhanced since we have made the density a bit more certain by even a weak measurement.) This is manifest in faster power-law decays: for example, $e^{i\theta}$ now scales as $x^{-1/K}$, i.e. decays twice as fast as in the unmeasured system.

The second set of results examines the more general situation with an ensemble of outcomes. By considering suitably nonlinear observables using the replica field theory, the authors demonstrate that there is a phase transition as a function of K at the level of this ensemble. Since the 'no click' outcome is a very special tail state in this ensemble, the natural expectation is that the ensemble transition occurs at some critical $K < 1$. The authors establish a critical value of $K = 1/2$ if the average over the ensemble is performed with the Born weights, but introducing a bias towards outcomes with a subextensive number of clicks (that are thus in a quantifiable sense 'near' the no-click outcome) then the transition shifts back to $K = 1$ — essentially capturing the stability of the $K = 1$ transition to weak disorder à la the Harris criterion [5].

A final idea introduced by the authors is a possible route to evading postselection, which requires some explanation. Naively, instead of postselection one could instead try to 'average over the ensemble' by running the experiment multiple times and summing over the results, since each outcome m appears with probability p_m automatically by the Born rule. Recall that ensemble-averaged quantities that track the transition are must be nonlinear in expectation values, e.g. $\sum_m p_m \langle O \rangle_m^2$ where O is some observable. Why not just use multiple runs to estimate this sum? To see why this fails, consider a single run of the experiment that prepares $|\psi_m\rangle$. Measuring observable O from this experiment produces an estimate O_m^{est} that must be an eigenvalue of O. Squaring this eigenvalue and averaging over many runs just produces the average of $\langle O^2 \rangle_m$ over the ensemble of outcomes, rather than the desired average of $\langle O \rangle_m^2$. So the authors propose a different trick: they sum over many runs, but only after weighting $\langle O \rangle_m$ by $\langle O \rangle_{m,C}$, which is an expectation value produced from a classical simulation. One can then compare two quantities: a 'quantum-classical' probe $E_{QC}[O] = \sum_m p_m \langle O \rangle_m \langle O \rangle_{m,C}$ obtained by running the experiment multiple times and averaging with this reweighting, and a 'classical-classical' probe $E_{CC}[O] = \sum_m p_m \langle O \rangle_{m,C}^2$. The authors argue that a coincident transition with K in *both* probes signals a nontrivial restructuring of the quantum ground state. The overhead of implementing this approach — which evades post-selection at cost of a classical simulation, and hence in essence pits the complexities of these two problems against each other — will depend on the system under study and may require various tricks such as coarse graining. Nevertheless, the idea is an intriguing one, and elucidating its complexity in various cases is an important goal.

A variation in this work, compared to many other studies of measurement-induced critical phenomena, is that the tuning parameter is not the rate of measurement, which is fixed. Rather, the transition in the post-measurement state is capturing a qualitative change in the nature of the underlying quantum critical system, by probing its correlations in a nontrivial way. While specific correlations that change in this example — such as the density — can be accessed by other more conventional techniques, it is intriguing to ask if the presence of subtle and hard-to-measure correlations, such as those encoding topological properties, can similarly trigger qualitative changes upon measuring simpler observables. If so, this might provide a new window into complex quantum-mechanical ground states. Given the emergence of a variety of new platforms such as Rydberg atom arrays that combine the analog simulation capabilities of traditional solid state systems with the programmability and measurement ability of digital quantum devices, it would appear that the time is ripe to consider such questions seriously. The present paper is a welcome first step in this direction.

References

[1] See M. Ippoliti and V. Khemani Phys. Rev. Lett. **126**, 060501 (2021) for one example of a post-selection-free proposal.

[2] T. Giamarchi, *Quantum Physics in One Dimension*, International Series of Monographs on Physics (Clarendon, New York, 2003).

[3] See, e.g. H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems.* (Clarendon, Oxford, 2002).

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[5] A. B. Harris, J. Phys. C **7**, 1671 (1974).