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The Corbino Paradox for Hydrodynamic Electrons

1. **Freely Flowing Currents and Electric Field Expulsion in Viscous Electronics**
   Authors: Michael Shavit, Andrey Shytov, and Gregory Falkovich
   Phys. Rev. Lett. **123**, 026801 (2019)

2. **Imaging hydrodynamic electrons flowing without Landauer–Sharvin resistance**
   Authors: C. Kumar, J. Birkbeck, J. A. Sulpizio, D. Perello, T. Taniguchi, K. Watanabe, O. Reuven, T. Scaffidi, Ady Stern, A. K. Geim and S. Ilani
   Nature **609**, 276 (2022)

3. **Spread and erase - How electron hydrodynamics can eliminate the Landauer-Sharvin resistance**
   Authors: Ady Stern, Thomas Scaffidi, Oren Reuven, Chandan Kumar, John Birkbeck, and Shahal Ilani
   arXiv: 2110.15369 (2021)

Recommended with a Commentary by Aaron Hui and Brian Skinner, Department of Physics, Ohio State University

Our thinking about electron transport is usually rooted in Ohm's law, $\mathbf{J}(x) = \sigma(x)\mathbf{E}(x)$, which is a conveniently local relation between the current density $\mathbf{J}(x)$ and the electric field $\mathbf{E}(x)$ at some point x. Underlying Ohm's law is an assumption that the rate of momentum relaxation, and therefore of energy dissipation, is determined by spatially local processes. Even though we often analogize the flow of electric current to the flow of water, such locality is not generally how actual fluids behave. Instead, fluids typically relax their momentum by viscous shearing, which arises from *gradients* of current density. Thus, viscosity generates a "nonlocality" in transport, where the local electric field depends on gradients of the current. Where viscous effects exist, the linearized equation of motion (for steady-state, incompressible flow) becomes

$$\left(\frac{1}{\sigma(x)} - \frac{\eta}{n^2e^2}\nabla^2\right)\mathbf{J}(x) = \mathbf{E}(x). \tag{1}$$

Here, η is the dynamical shear viscosity. One can think of the viscous term in Eq. (1) as the next-leading term in a gradient expansion for the resistivity. In the electron context,

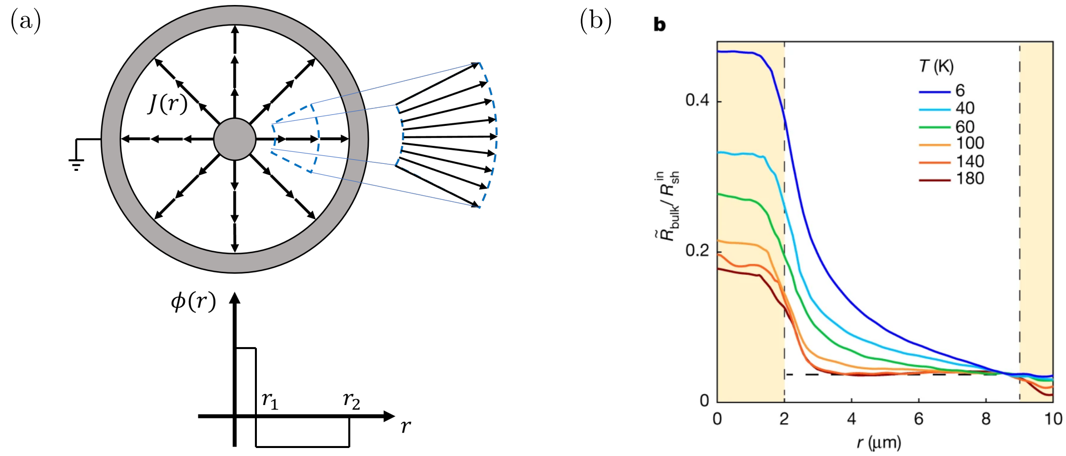

Figure 1: (a) Current flow in the Corbino geometry. The flow is rotationally symmetric, with a flat electric potential profile in the bulk and sharp voltage jumps at the contacts. As shown in the blow-up, the flow has shear gradients, implying that energy is being dissipated via viscosity. This figure is adapted from Fig. 2 of the first recommended paper. (b) The experimentally measured electric potential as a function of distance from the center of the Corbino geometry, as reported in the second recommended paper. As the temperature is increased and the mean free path for electron-electron scattering is reduced, the electric potential becomes increasingly flat.

the viscous term arises from electron-electron collisions. This term describes the diffusion of electric current, where η can be thought of as a microscopic diffusion constant (kinematic viscosity) multiplied by the electron mass density.

Excitingly, recent developments have shown strong evidence of a hydrodynamic regime for graphene electrons (see, e.g., the Oct. 2015 JCCMP article by Guinea and the papers it recommends, or the reviews in Refs. [1, 2]). Graphene is particularly suitable for realizing electron hydrodynamics because of its weak electron-phonon coupling and the relatively low disorder of clean graphene samples; these factors reduce the relative importance of non-viscous dissipation. Experimental evidence for hydrodynamic behavior in graphene has come in multiple forms during the past six years, such as electron backflow [3], Wiedemann-Franz law violations [4], and "superballistic" conductance larger than the Landauer-Sharvin value [5]. But the first two papers recommended here consider a perhaps more surprising and paradoxical feature of hydrodynamic electron flow, which we refer to as the "Corbino Paradox."

Consider the flow of electric current in a radially-symmetric Corbino (annular) geometry, shown in Fig. 1a. The Corbino geometry has the nice feature of having no lateral edges, and therefore there is no need to worry about boundary conditions other than those at the contacts. (The nature of the lateral boundary conditions – e.g., no-slip or no-stress – is highly important for hydrodynamic behavior in the rectangular geometry; see e.g. [6].) In fact, in the Corbino case the solution for the current density follows immediately from current conservation and radial symmetry:

$$\mathbf{J}(\mathbf{x}) = \frac{I}{2\pi r}\hat{\mathbf{r}}, \tag{2}$$

where I is the net current. If one now uses Equation (1) to calculate the electric field, one can see that something strange happens in the limit of purely hydrodynamic behavior (no scattering of electrons by impurities or phonons), where $\sigma \to \infty$. Since the radially symmetric solution has $\nabla^2 \mathbf{J} = 0$, we find that $\mathbf{E}(x) = 0$ everywhere! In other words, the current flows without forcing.

The existence of finite current with zero electric field may not seem overly troubling since we have taken the limit $\sigma \to \infty$. But the "paradox" is that our radially-symmetric solution for the current *does* involve the dissipation of energy. As the current flows outward, it experiences shear stresses (see Fig. 1a), which means there is a dissipated power throughout the bulk of the device with a magnitude proportional to the shear viscosity. Thus we arrive at the apparent "Corbino paradox": a flow of current without any driving electric field that still dissipates energy. Is this a hydrodynamic perpetual motion machine?

This seeming paradox is posed and resolved in the first recommended paper by Shavit, Shytov, and Falkovich. The power P dissipated during viscous shearing is of course ultimately injected by the power source that drives the current, $P = IV$. The authors' key idea is that, since the electric field is zero within the bulk of the device, there must be a sharp voltage jump $\Delta\phi$ at the interface between the bulk and a given contact. The authors derive this voltage jump as

$$\Delta\phi = 2\eta K \mathbf{v} \cdot \hat{\mathbf{n}}, \tag{3}$$

where $\mathbf{v}$ is the fluid velocity, $\hat{\mathbf{n}}$ is the unit normal vector at the contact edge, and K is the signed extrinsic curvature of the boundary. For a circular annulus, the curvature $K = 1/r$, so that the total resistance for the Corbino geometry in the hydrodynamic limit is

$$R = \frac{\eta}{\pi n^2 e^2} \left[\frac{1}{r_1^2} - \frac{1}{r_2^2} \right], \tag{4}$$

where r_1 and r_2 are the radii of the inner and outer contacts, respectively.

Thus the authors arrive at striking set of predictions that stand in sharp contrast to usual Ohm's law thinking: hydrodynamic flow in the Corbino geometry exhibits bulk electric field expulsion and a sharp viscous "contact resistance." Unlike in Ohmic flow, the input power and the energy dissipation are spatially separated; the former occuring at the contacts and the latter throughout the bulk.

These predictions seem to be directly verified in the second recommended paper by Kumar et al. The authors use a scanning single-electron transistor technique (based on measuring the Coulomb blockade through a single carbon nanotube hovering just above the sample [7]) to map out the electric potential as a function of position across a graphene sample with Corbino contacts. The result is shown in Fig. 1b. The crossover from a "ballistic" regime (where electrons effectively do not scatter) to a "hydrodynamic" regime is controlled by increasing the temperature, which raises the rate of electron-electron scattering. When the temperature is high enough, the electron-electron mean free path becomes much shorter than the sample dimensions, and the resulting hydrodynamic flow leads to a flat profile of electric potential in the bulk of the sample.

This charming set of results may nonetheless leave a reader who is trained in quantum condensed matter feeling slightly uneasy. It is common to teach students that the conductance in a quantum system has a fundamental upper limit equal to e^2/h multiplied by the number of conducting channels (i.e., the number of transverse quantum states that are occupied). This is the well-known Landauer-Sharvin limit of conductance, and it seems to not be respected by Eq. (4). Indeed, as is discussed nicely by Kumar et. al., in a Corbino geometry the number of conducting channels increases with increasing radius, leading to an electric potential that decays as $\sim 1/r$ in the ballistic limit. How do hydrodynamic electrons manage to circumvent the Landauer-Sharvin limit of conductance?

This latter question is the focus of the third recommended paper by Stern et. al. The authors construct a description using semiclassical Boltzmann theory in which they are able to tune the strength of electron-electron interactions to pass continuously from the ballistic limit to the hydrodynamic limit. They focus on a somewhat different geometry, but nonetheless are able to demonstrate the appearance of the same qualitative behavior as one enters the hydrodynamic regime: field expulsion and a sharp contact resistance. Intuitively, the Landauer-Sharvin limit is surpassed because electron-electron collisions allow electrons to diffuse easily between different conducting channels. In this way electrons readily jump from channels that are ending (as, say, the geometry constricts) to channels that are transmitted, like an old-time freighthopper traveling across the country by jumping from one train to another.

One interesting outstanding question is about the nature of the contact-bulk interface. The fluid solution suggests that the potential drops across an interfacial layer (Knudsen boundary layer) whose thickness is comparable to the electron-electron mean free path. But how, exactly, is the electric potential being screened in such a way that it reflects the flow throughout the bulk? And why does the sign of the voltage jump at the contact change for concave versus convex boundaries? Shavit et. al. offer some words about relevant interfacial physics, but it seems that a comprehensive theory still does not exist, and the two different signs of voltage jump are not clearly seen by the experiments of Kumar et. al. It is also worth mentioning that while Kumar et. al. seem to have observed the field expulsion associated with hydrodynamic behavior, the universal viscous contact resistence predicted in Equation (4) has yet to be verified experimentally.

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