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Anomalous quantum anomalous Hall effect

1. **Experimental signature of the parity anomaly in a semi-magnetic topological insulator**
   Authors: M. Mogi, Y. Okamura, M. Kawamura, R. Yoshimi, K. Yasuda, A. Tsukazaki, K. S. Takahashi, T. Morimoto, N. Nagaosa, M. Kawasaki, Y. Takahashi, and Y. Tokura
   Nature Physics **18**, 390 (2022) — arXiv:2105.04127

2. **Half-quantized Hall effect and power law decay of edge-current distribution**
   Authors: J.-Y. Zou, B. Fu, H.-W. Wang, Z.-A. Hu, and S.-Q. Shen
   Phys. Rev. B **105**, L201106 (2022) — arXiv:2202.08493

Recommended with a Commentary by Carlo Beenakker, Instituut-Lorentz, Leiden University

In the context of the quantum Hall effect one identifies two anomalies: the *quantum anomalous Hall effect* and the *anomalous quantum Hall effect*. The former is Haldane's quantum Hall effect without Landau levels: The quantization of the Hall conductance follows from the coupling of a magnetic moment to the electron spin, without any orbital effect from the Lorentz force. The latter is the half-integer quantum Hall effect observed in graphene by Geim and Novoselov: Because of the spin and valley degeneracy one would expect the first Hall plateau at $4e^2/h$, but instead it is at half that value.

The publication by Mogi *et al.* reports on a combination of these two anomalies. Their objective is to measure a Hall conductance σ_{xy} equal to $\frac{1}{2}e^2/h$ in zero magnetic field, without any degeneracy factor. The Hamiltonian that would produce this result is simple,

$$H = v(p_x\sigma_x + p_y\sigma_y) + M\sigma_z, \tag{1}$$

it's the Hamiltonian of a two-dimensional Dirac fermion (momentum $\boldsymbol{p}$, velocity v), with its spin $\boldsymbol{\sigma}$ coupled to an out-of-plane magnetization M. The energy spectrum $E = \pm\sqrt{v^2p^2 + M^2}$ has a gap $2M$ centered at $E = 0$. The Hall conductance

$$\sigma_{xy} = \frac{1}{2}\frac{e^2}{h}\,\mathrm{sign}\,M \tag{2}$$

depends only the sign of the magnetization, as long as the Fermi level is inside the gap.[1]

[1] The dependence of the sign of the Hall conductance on wether the magnetization points upwards or downwards in the plane is referred to as a "parity anomaly".

The physical realization is a thin film of a three-dimensional topological insulator (3D TI, a $(\text{Bi,Sb})_2\text{Te}_3$ alloy with a large bulk gap $\approx 200\,\text{meV}$), doped by magnetic ions (Cr) on the top surface only. (See Fig. 1.) Dirac fermions exist on the top and bottom surfaces, gapless on the bottom and with a gap $2M \approx 30\,\text{meV}$ on the top. The film is sufficiently thick ($d \approx 10$ nm) that the two surfaces are uncoupled.

Earlier experiments (discussed in a 2013 Journal Club contribution) on a 3D TI with a homogenous magnetic doping, so that both surfaces are gapped, had measured the quantum anomalous Hall effect: a quantization of the Hall conductance at e^2/h in zero external magnetic field. A 3D TI with a single gapped surface is referred to as a *semi-magnetic topological insulator*. Last year a realization was reported, but without experimental data for the Hall conductance.

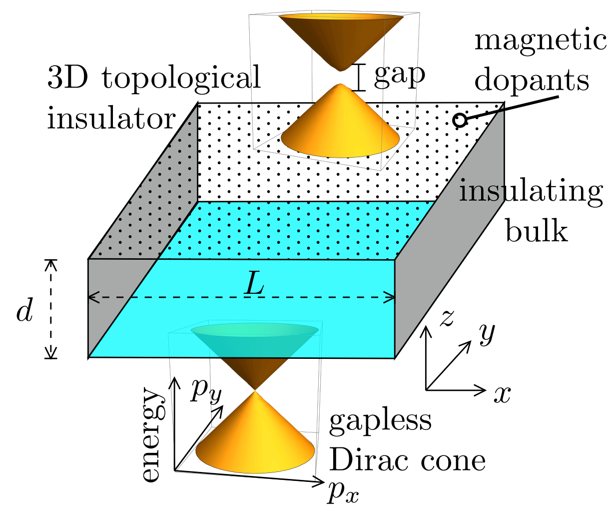

Fig. 1. Semi-magnetic topological insulator.

The two surfaces contribute in parallel to the Hall conductance, by gapping out one surface one would expect to obtain the anomalous half-integer value. Mogi *et al.* indeed report this effect, but not in the way familiar from the integer quantum Hall effect. There are no dissipationless chiral edge states in a 3D TI, instead, a dissipative current is carried by the gapless bottom surface.

The current distribution has been calculated by Zou *et al.*, in a simple model where the gapped upper surface is represented by a boundary condition on the wave function ψ in the gapless lower surface, of the form

$$\sigma_y \psi(\pm L/2, y) = \pm\psi(\pm L/2, y). \tag{3}$$

This boundary condition corresponds to the $M \to \infty$ limit of the gap in the upper surface. The infinite mass limit allows for an analytical result for the current density profile,

$$\begin{aligned} j_y(x) &= f(x + L/2) - f(L/2 - x), \\ f(x) &= \frac{e\mu}{2h} x^{-1} J_1(2\mu x/\hbar v), \end{aligned} \tag{4}$$

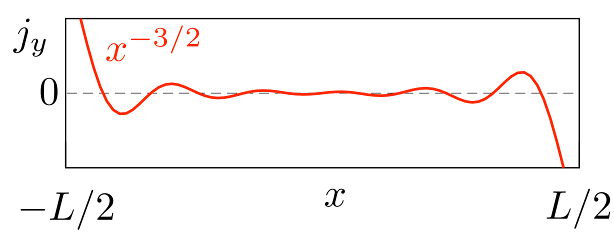

Fig. 2. Current density in the gapless bottom surface of the 3D TI, calculated by replacing the effect of the gapped upper surface by an infinite-mass boundary condition at $x = \pm L/2$. The current decays $\propto x^{-3/2}$ away from the edge. Figure from Zou *et al.*

with J_1 a Bessel function. The current density (plotted in Fig. 2) includes the contributions from all states on the lower surface with energy smaller than the Fermi energy μ. The current density is peaked at the edge, with an algebraic $x^{-3/2}$ decay — rather than the exponential decay one has in the integer quantum Hall effect. The total current $I_\pm$ carried by the edge at $x = \pm L/2$ equals

$$I_\pm = \pm \int_0^\infty f(x)\, dx = \pm \frac{e\mu}{2h}. \tag{5}$$

A voltage difference $\Delta\mu = eV$ between opposite edges (in the x-direction) then gives the current $(e^2/2h)V$ in the y-direction, and hence the half-integer Hall conductance $\sigma_{xy} = e^2/2h$.

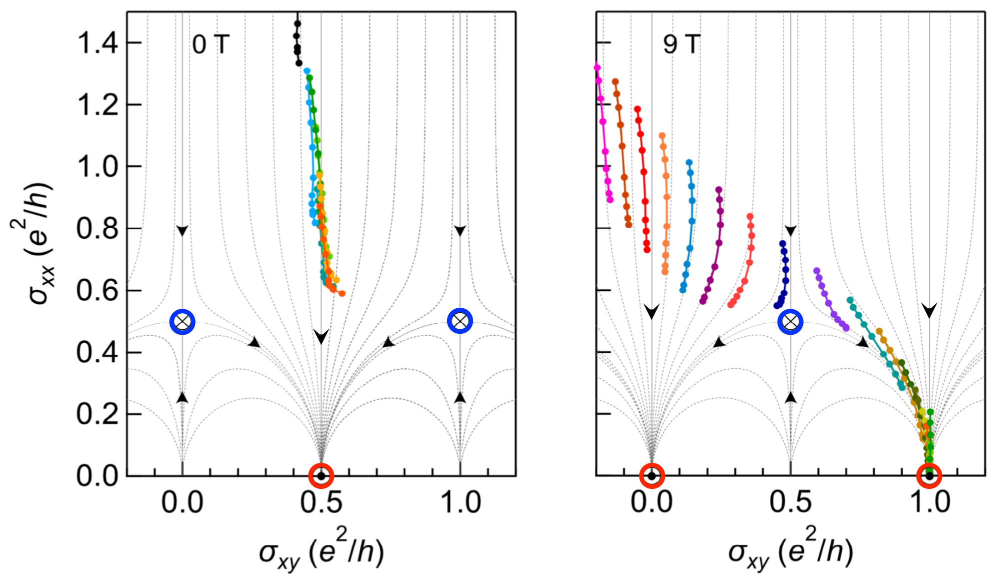

Fig. 3. Longitudinal conductance σ_{xx} and Hall conductance σ_{xy} of the semi-magnetic topological insulator of Fig. 1, obtained by inverting the resistance tensor measured in a Hall bar geometry. The left panel is in zero magnetic field, the right panel in a high magnetic field perpendicular to the thin film. The experimental data is superimposed on the temperature flow diagram of the integer quantum Hall effect, displaced by one half e^2/h in the left panel. Red and blue circles indicate stable and unstable fixed points of the flow. The data points of a given color flow in the direction of the arrows when the temperature is decreased from 0.7 K down to 0.04 K. Different colors distinguish different gate voltages, used to vary the Fermi energy on the lower surface. (The upper surface remains gapped in the entire gate voltage range.) Figure from Mogi *et al.*

The experimental results of Mogi *et al.* are summarized in Fig. 3 (left panel). Because the lower surface is not gapped, the edge currents are not protected against backscattering, and indeed the longitudinal conductance σ_{xx} does not vanish. What is remarkable is that the Hall conductance deviates only little from the ideal value $e^2/2h$ which would appear in the absence of backscattering.

The same figure also shows the conductance which is measured when a large perpendicular magnetic field is applied perpendicular to the thin film. That data (right panel) shows the expected flow with decreasing temperature to $\sigma_{xx} = 0$, $\sigma_{xy} = e^2/h$. Notice that the quantization of σ_{xy} is only reached when $\sigma_{xx} \approx 0$ in that case. One might have expected the flow diagram in the half-integer quantized case to be simply the one from the integer quantization displaced by half a conductance quantum. The experiment shows a qualitatively different flow. I do not have a convincing explanation.