# Flocking without moving

**Nonmutual torques and the unimportance of motility for long-range order in two-dimensional flocks**
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*Recommended with a Commentary by Alexandre Solon, Sorbonne Université and CNRS*

It has become well appreciated that most (if not all) examples of active matter show violations of Newton's third law of action-reaction at the scale at which we observe them. This is obvious when considering macroscopic animals that have "social" interactions (say birds in a flock or humans in a crowd) but also expected for microscopic active particles. For example two self-propelled colloids may each interact reciprocally with a solvent, abiding Newton's action-reaction law, but the effective interaction between the two colloids will in general be non-reciprocal since out of equilibrium the interaction has no reason to derive from a potential energy.

Some generic consequences of the non-reciprocal interactions between two species of particles have been recently explored. In particular, two diffusing species with non-reciprocal couplings can exhibit spontaneous symmetry breaking in the form of traveling patterns [1, 2] while two species with vectorial order parameters, such as aligning self-propelled particles, spontaneously break chiral symmetry and start to rotate, which happens at a transition that is formally similar to the exceptional points of non-Hermitian quantum mechanics [3]. Non-reciprocal interactions between individuals of the same species have also been considered, primarily under the form of a vision cone: particles pointing in a given direction tend to react more strongly to the ones that they see in front of them. This leads to new patterns for particles with attractive interactions [4] and a qualitatively different phase diagram in a Vicsek-like model of aligning particles [5].

To my knowledge, the work of Dadhichi *et al.* is the first to clearly spell out the consequences of the vision-cone type non-reciprocal interactions at the level of the hydrodynamic equations describing the large scale behaviour of these systems. To do this, they start from a microscopic model in 2d in which particles self-propel at constant speed $v_0$ in the direction $\hat{\mathbf{v}}_\mathbf{i}$ (for particle $i$) so that their velocity is $\mathbf{v}_i = v_0\hat{\mathbf{v}}_\mathbf{i}$. They include rotational inertia, each particle then has in addition a "spin" angular momentum $\mathbf{s}_i$ along an axis $\hat{\mathbf{z}}$ orthogonal to the 2d plane in which the particles live, which rotates the velocity $\dot{\hat{\mathbf{v}}}_\mathbf{i} = \frac{1}{\chi}\mathbf{s}_\mathbf{i} \times \mathbf{v}_i$. The non-reciprocal interaction appears in the dynamics of the angular momentum which reads

$$\dot{\mathbf{s}}_\mathbf{i} + \frac{\eta}{\chi}\mathbf{s}_\mathbf{i} = \sum_{\mathbf{r}_j \in \mathcal{R}_i} [J + A(\hat{\mathbf{v}}_\mathbf{i} \cdot \hat{\mathbf{r}}_\mathbf{ij})]\, \hat{\mathbf{v}}_\mathbf{i} \times \hat{\mathbf{v}}_\mathbf{j} \tag{1}$$

where $\eta$ is a friction due to the surrounding medium and $\mathcal{R}_i$ denotes the neighbourhood of particle $i$ and $\hat{\mathbf{r}}_{\mathbf{ij}}$ the unit vector in the direction $i \to j$. The non-reciprocity comes from the term with coefficient $A$ which, for positive $J$ and $A$, increases the strength of alignment in front of particle $i$ and decreases it in the back.

Coarse-graining the dynamics, Dadhichi *et al.* uncover two main effects of the non-reciprocity at the level of continuous equations. First, after integrating out the spin, which is a fast variable, they find that the dynamics of the velocity field has an advection term $(\mathbf{v} \cdot \nabla)\mathbf{v}$ proportional to $A$, in addition to the one due to self-propulsion. Because of this mechanism, the fluctuations of the velocity field propagate at a different speed from density fluctuations which are advected at speed $v_0$. The consequences could be important for the phenomenology observed and are not fully understood. Interestingly, the new advection term is also expected in absence of self-propulsion and can even play the role of the latter in a system where particles are not moving, hence the title of the paper of Dadhichi *et al.* and of this commentary.

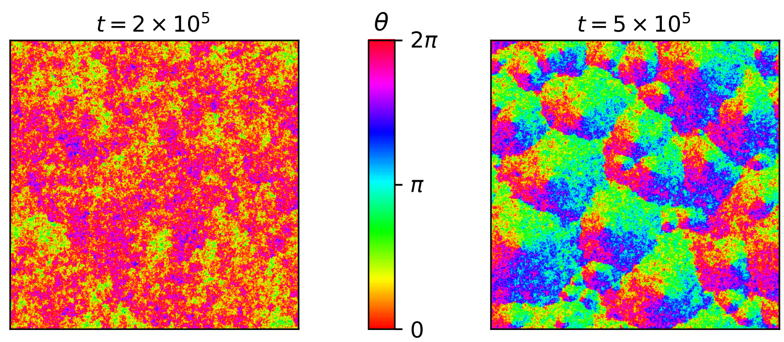


Figure 1: Simulation of the non-reciprocal XY model described in the text with $\varepsilon = 0.6$, $T/J = 0.6$ and system size $400 \times 400$ starting from an ordered initial condition.

An illustration that is directly suggested by the authors is shown in Fig. 1. It features a non-reciprocal XY model (NRXY) [6] where XY spins parameterized by an angle $\theta_i$ are fixed at the vertices of a square lattice and interact in a way akin to Eq. (1) (except without inertia)

$$\frac{d\theta_i}{dt} = -\sum_{j \in \mathcal{N}_i} J\left(1 + \varepsilon \hat{\mathbf{u}}_{\mathbf{i}} \cdot \hat{\mathbf{r}}_{\mathbf{ij}}\right) \sin(\theta_i - \theta_j) + \sqrt{2T}\eta_i \tag{2}$$

with $\eta_i$ a delta-correlated Gaussian white noise, $\hat{\mathbf{u}}_{\mathbf{i}} = (\cos\theta_i, \sin\theta_i)$, $\mathcal{N}_i$ the set of nearest neighbours of site $i$ and $T$ a noise strength which is the temperature when the system is in equilibrium at $\varepsilon = 0$. At low $T/J$ and non-zero $\varepsilon$, one observes a long-ranged ordered phase which is metastable and gives way to a dynamical foam of topological defects at long times. This is exactly the behaviour observed for flocks at constant density [8], the field theory that describes “Malthusian flocks” composed of particles that divide and die on a fast time scale [9]. Fig. 1 thus confirms the findings of Dadhichi *et al.* that the non-reciprocity by itself generates, even for fixed particles, what is usually described as flocking.

The second effect of non-reciprocity highlighted by Dadhichi *et al.* is that it decreases the longitudinal diffusion coefficient. The responsible term is proportional to $\chi A^2$ and thus becomes stronger when the inertia and the non-reciprocity increase. If large enough, the longitudinal diffusion turns negative which signals a small wavelength instability. They show that the relation dispersion of the unstable modes is non-trivial, with a finite band of most unstable modes. This effect seems different from the (transverse) instability seen in a front-back asymmetric Vicsek model [5]. It would thus be interesting to search for it and, beyond the linear stability, for the nonlinear steady-state in which the system would settle.

The work of Dadhichi *et al.* shows that the case of polar particles with a front-back anisotropy is already quite rich. It opens the way to investigating other symmetries using microscopic models and coarse-grained hydrodynamic description. In addition, the main finding that the front-back anisotropy acts essentially in the same way as activity in advecting the order parameter could be important in designing new types of active matter that do not move.

# References


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