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Quantum Oscillations without Landau Quantization

Kagomé Quantum Oscillations in Graphene Superlattices
Authors: Folkert K. de Vries, Sergey Slizovskiy, Petar Tomić, Roshan Krishna Kumar, Aitor Garcia-Ruiz, Giulia Zheng, Elías Portolés, Leonid A. Ponomarenko, Andre K. Geim, Kenji Watanabe, Takashi Taniguchi, Vladimir Fal'ko, Klaus Ensslin, Thomas Ihn and Peter Rickhaus
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Recommended with a Commentary by Nigel Cooper, University of Cambridge

The observation of quantum oscillations in a metal – the periodic modulations of physical properties with the inverse magnetic field – is one of the primary ways to identify and characterize the Fermi surfaces of metals [1]. These oscillations arise from the Landau quantization of the orbits of low-energy quasiparticles in the metal, imprinting field-dependent energy spacings (set by the cyclotron energy) into the density of states which lead to oscillations in thermodynamic and transport properties. The analysis of these oscillations [1, 2] provides detailed information on the Fermi surface geometry and the properties of the low-energy quasiparticles. The prevalence of new classes of materials – including topological insulators and semimetals, and van der Waals materials – has initiated a re-exploration of aspects of quantum oscillations, even in regimes where the physics is dominated by single-particle effects [3, 4, 5, 6, 7].

In the highlighted paper, it is shown experimentally and theoretically that the Moiré potential in bilayer graphene leads to a novel effect in the quantum oscillations of the conductivity, which the authors refer to as "Kagomé oscillations" for reasons outlined below. These Kagomé oscillations have rather different characteristics from the conventional Shubnikov–de Haas (SdH) oscillations of the conductivity. In particular, the Kagomé oscillations persist to high temperatures: unlike standard quantum oscillations, they are not suppressed when the Fermi surface is thermally smeared by more than the cyclotron energy and so do not rely on Landau quantization of the energies. The paper puts forward an explanation that connects to special features of the geometry induced by the Moiré potential. The relevant symmetry generalizes to other cases, leading to the prediction of Kagomé oscillations in other settings.

The phenomenon is perhaps most clearly introduced by presenting the experimental data itself, see Fig. 1. These data are taken on a bilayer graphene sample, with small interlayer twist, $\simeq 1.9°$, which creates a hexagonal Moiré superlattice with a relatively long period, corresponding to a mini-band Brillouin zone (BZ) of small reciprocal space area $\mathcal{A}$.

Figure 1(a) shows the field- and density-dependence of the longitudinal conductivity at low temperatures. The strongest features are the sets of SdH oscillations arising from the fans of Landau levels that emerge from the charge neutrality point $n = 0$ and from the two points of filled mini-bands at $n = \pm n_{\rm MB}$ where $n_{\rm MB} \simeq 8.9 \times 10^{12}{\rm cm}^{-2}$. These SdH oscillations can be understood with reference to the dispersion relation for the mini-bands formed by the Moiré potential. [See inset to Fig. 1(a).] The Landau levels formed around $n = 0$ (arrows in blue) are related to quantization of the (near circular) electron-like orbits around the κ and κ' points in the mini-band BZ. The Landau levels formed around $n = n_{\rm MB}$ (arrows in red) arise from the hole-like orbit that encircles the γ point of the mini-band BZ.

As the temperature is increased, these SdH oscillations are expected [2] to be suppressed as $\sigma_{xx} \propto X/\sinh(X)$ with $X \equiv 2\pi^2 k_B T/(\hbar\omega_c)$ with $\hbar\omega_c$ the cyclotron energy (of the order of a few Kelvins for the field scales here, $B \lesssim 6$T). Indeed, this is found to be the case. However, surprisingly, there are other oscillations that survive to much higher temperatures.

Figure 1(c) shows the field-and density-dependence of the longitudinal conductivity at a high temperature, where the conventional SdH oscillations have been suppressed by thermal smearing. There remain oscillations that are approximately horizontal in Fig. 1(c), i.e. oscillations in field B but not in density n.

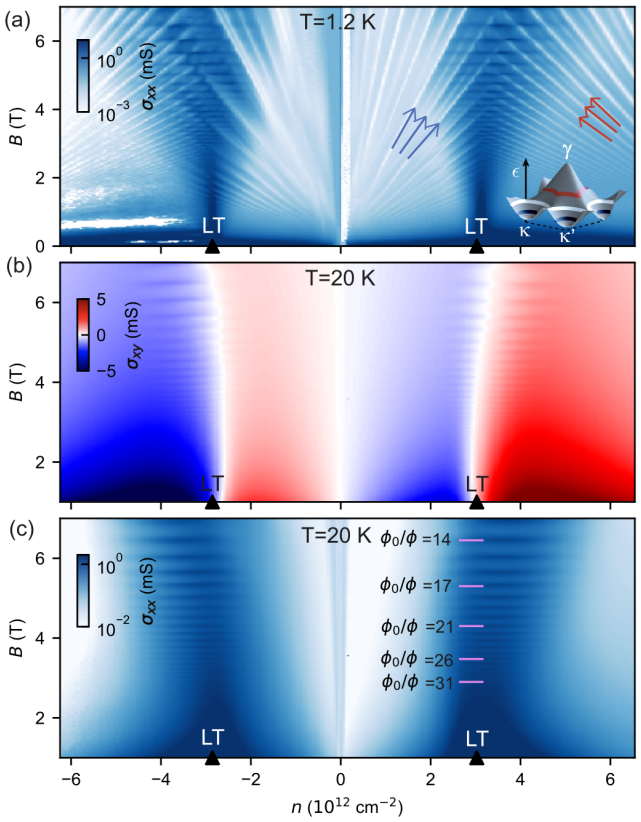

Figure 1: Magnetotransport oscillations in a twisted bilayer graphene device as a function of density n and magnetic field B. (a) Longitudinal conductivity at low temperatures, showing conventional SdH oscillations. (b) Hall conductivity at high temperatures, showing the transition from electron-like to hole-like transport at the Lifshitz transition for the Moiré mini-band as the regions of white. (c) Longitudinal conductivity at high temperatures, showing the Kagomé oscillations in the vicinity of the Lifshitz transition. [From de Vries *et al.*, arXiv:2303.06403]

The paper refers to these oscillations as Kagomé oscillations and explains how they can arise from the quantum interference of a class of trajectories for electrons moving through the Moiré superlattice. The relevant trajectories are those at, or close to, the energy corresponding to the Lifshitz transition (LT), at $n \simeq n_{\rm LT}$, between the electron-like trajectories ($0 < |n| < n_{\rm LT}$) and the hole-like trajectories ($n_{\rm LT} < n < n_{\rm MB}$). The transition between these two regimes can be seen as the line in the $n - B$ diagram where the Hall conductivity, σ_{xy}, changes sign [white regions in Fig. 1(b)]. The momentum-space view of the interference responsible for these oscillations is illustrated in Fig.2, which shows the mini-BZ in

the vicinity of one of the Dirac points [i.e. another view of the inset to Fig. 1(a)]. The energy contour at the LT is shown in black, forming an extended network that appears like a distorted Kagomé lattice. (The distortions respect an underlying C_3 rotation symmetry.) The interference responsible for the Kagomé oscillations arises from the two possible paths in reciprocal space to get from an initial state $|\text{in}\rangle$ to a final state $|\text{out}\rangle$. [As is standard, within the semiclassical picture for the dynamics of an electron in a magnetic field B, the real-space trajectories can be obtained from the reciprocal-space paths by rescaling by a factor of $\ell_B^2 = \hbar/(eB)$ and rotating by $\pi/2$. Thus the real-space interference for electrons at the energy of the LT also occurs between trajectories that run over a Kagomé lattice.]

Crucial to the qualitative features of the Kagomé oscillations, the two interfering trajectories in Fig. 2 combine an equal number of equivalent sections: the four sections of the (upper) green path are in one-to-one correspondence with each of the four sections of the (lower) blue path. Thus the dynamical phases accumulated on the two paths are equal, independent of the precise energy of the electron. This contrasts strongly with conventional Landau-quantized orbits, for which the phase accumulated within a closed loop changes by 2π as the energy increases by the cyclotron energy. Here, for the Kagomé oscillations, there is no change in phase as a function of energy, so thermal smearing of the electron energy does not affect the interference. The energy of the electron is relevant only with regard to the transmission probabilities at the saddle points of the energy dispersion (the crossing points of the black lines in Fig. 2). For an energy that is very large (very small) compared to the LT energy, the electron will follow only the hole-like (electron-like) trajectories and no Kagomé interference will occur. This accounts for the suppression of the Kagomé oscillations for large $|n - n_{\text{LT}}|$.

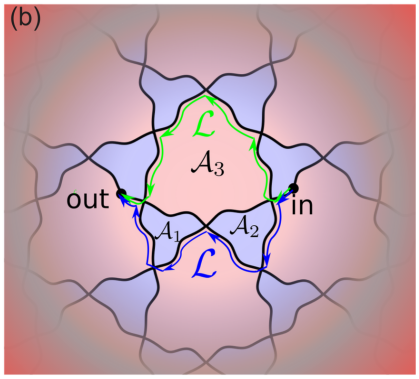

Figure 2: Sketch of the Brillouin zone of the mini-bands arising from the Moiré superlattice. The black line indicates the equipotential contour corresponding to the Lifshitz transition. Interference between the green and red trajectories leads to Aharonov Bohm oscillations with $\Delta(1/B) = 2\pi e/(\hbar\mathcal{A})$ where $\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3$ is the total area of the mini-band BZ. [From de Vries *et al.*, arXiv:2303.06403]

These two paths enclose an area which covers the full area of the mini-band BZ, $\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3$. Thus, the periodicity of the Aharonov-Bohm oscillations resulting from the interference of the two trajectories is $\Delta(1/B) = 2\pi e/(\hbar\mathcal{A})$. This matches the expected periodicity for the formation of (a subset of) the Brown-Zak mini-bands for electrons moving in a magnetic field in this periodic potential, which build up the associated Harper-Hofstadter spectrum: these form when $q \times (eB/h) \times A = p \times \phi_0$ with p and q integers, with $A = (2\pi^2)/\mathcal{A}$ the area of the real-space cell and $\phi_0 = h/e$ the flux quantum. The authors draw a distinction from that process, arguing that the Kagomé oscillations are less strongly suppressed by scattering than the Brown-Zak mini-bands. Indeed, signatures of those mini-bands had been seen previously in regimes of stronger magnetic field [8, 9], and were found to lead to rather different physical consequences, extending to large densities $n \gtrsim n_{\text{LT}}$. The Kagomé oscillations, being tied to the LT, $n \simeq n_{\text{LT}}$, therefore appear to be in a different physical regime.

The mechanism put forward is rather generic. Indeed, the authors present experimental results showing that similar Kagomé oscillations appear for the Moiré material formed from graphene on HBN in the density regime for the Lifshitz transition. Furthermore, recognizing that the cancellation of phases is related to the discrete rotational symmetry, the authors predict that similar oscillations should arise in materials with C_4 symmetry. It will be interesting to explore further the appearance of Kagomé oscillations in other materials and to understand their robustness to the various forms of scattering that can be at play in these materials. One can also wonder how, if at all, the Kagomé oscillations might appear in thermodynamic quantities such as the density of states or magnetization.

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