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Progress measuring fractional quantum numbers in quantum Hall interferometers

Fabry-Perot interferometry at the $\nu = 2/5$ fractional quantum Hall state
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Two years ago, we wrote [1] a Recommendation for a paper by Nakamura *et al*[2] reporting on an interferometric measurement of the fractional statistics of the quasi-particles in the $\nu = 1/3$ quantum Hall state. One of us (SAK) has contributed Recommendations[3] on earlier interferometric experiments with related goals by Camino *et al* and Willett *et al*. However, the physics involved is of considerable fundamental interest, and while the underlying theory is simple and compelling, there have remained unresolved issues of interpretation and worries about possible experimental artifacts [4]. Final certainty about the results will likely emerge from a web of evidence stemming from numerous experiments that extend the range of existing experiments, or check and confirm uncertain aspects of earlier ones.

Excitingly, there have been new developments on the problem of measuring fractional statistics, including the current recommended paper which reports an extension of the previous interferometer experiments, now extended to the case of the $\nu = 2/5$ quantum Hall state. As we will discuss below, this is an important extension of the previous result, one in which statistics, effective charge, and filling factor are not all the same. In addition, there have been other notable developments in the area of quantum Hall interferometers, to which we will also allude including a new form of interferommetry [5], an improved version of a prevously studied interferometer [6] and the construction of a set of graphene-based interferometers [7, 8, 9, 10] that promise to greatly expand the range of these experiments.

Early in the history of the fractional quantum Hall effect, a remarkable theoretical result was obtained by Arovas, Schrieffer, and Wilczek (ASW) [12] on the basis of an analysis of the Laughlin wave function [11]; if a quasi-hole is transported adiabatically along a closed path, the Berry phase evolved can be expressed in terms of the expectation value of the total charge, $\hat{Q}$, enclosed by the path as

$$\theta = -2\pi\langle\hat{Q}\rangle \ . \tag{1}$$

This result can be interpreted in several ways. ASW pointed out that if the path enclosed (but did not closely approach) an integer number n_{qh} of quasi-holes with charge e/q in an otherwise incompressible state with filling fraction $\nu = 1/q$, then

$$\theta = \theta_{AB} + n_{qh}\theta^{\star} \tag{2}$$

where $\theta_{AB} = 2\pi[\Phi/\Phi_0^{\star}]$ where Φ is the total magnetic flux enclosed, and $\Phi_0^{\star} = hc/e^{\star}$ is an effective flux quantum for a particle with charge $e^{\star} = e/q$, and $\theta^{\star} = 2\pi/q$ is the statistical phase that governs the braiding statistics - and thus identifies the quasi-particles as Abelian anyons. (Indeed, they have the same statistical phase invoked earlier by Halperin[13] in constructing heirarchical wave-functions for additional fractional quantum Hall states, starting with $\nu = 2/5$.) Alternatively, the same equation can be interpreted as reflecting an indication of particle-vortex duality, in which in the dual description charge becomes flux and flux becomes charge [14].

Later, it was proposed that θ - both as a measure of the fractional charge and of the fractional statistics - could be measured in a Fabry-Perot-like interferometer. [15] Importantly, it was also recognized that the nature of the Berry phases involved is necessarily more subtle (and still more interesting) for heirarchical states, [16] such as the $\nu = 2/5$ state, and even more so for states that support non-Abelian anyons, [17, 18] such as presumably the $\nu = 5/2$ or $\nu = 7/2$ states. However, achieving the requisite experimental control - especially control of Coulomb charging energy - meant that while relatively rapid progress was made in obtaining interferometric measures of the fractional charge [19], the measurement of fractional statistics is only just now being realized.

In the new experiment [20], an interferometer—a large ($1\mu m^2$) multi-electron (~ 1000) quantum dot with two quantum-point-contact leads—similar to the device investigated in Ref. [2], with the same proximal back gate grown into the heterostructure—was operated with the bulk regions inside and outside the dot at filling factor $\nu = 2/5$ as well as $1/3$ as in Ref. [2].

The point contacts were tuned to either fully or partially transmit edge modes from regions outside of the dot through to the inside of the dot. In this way, a comparison of three different regimes could be made in a single device: (i) the regime with $\nu = 1/3$ in the bulk and partial transmission of a single $1/3$ edge (the same configuration as in Ref. [2]); (ii) the regime with $\nu = 2/5$ in the bulk where only the outermost of the two edge modes (closest to the gates) is partially transmitted; and (iii) the regime with $\nu = 2/5$ in the bulk and only the innermost edge mode (farthest from the gates) is partially transmitted. In these last two cases it is tempting to identify the outer of the two edge modes as the "1/3 edge mode" and the inner as the "2/5 edge mode." The authors are careful to compare interferometry signals for a partially transmitted inner edge mode at $\nu = 2/5$ with signals from a partially transmitted single edge mode at $\nu = 1/3$ before making this identification. This is the natural association, but it could have been otherwise, that the two edges at $\nu = 2/5$ mixed and lost their identities.

In fact, interference of the outer mode at $\nu = 2/5$ and the single mode at $\nu = 1/3$ do show several differences that complicate a simple mapping. First, the gaps are smaller at $\nu = 2/5$, as the field is lower. Second, interference of the outer edge at $\nu = 2/5$ has a gapless inner edge inside the dot that can absorb quasiparticles. The lack of clear phase jumps—the key signature of fractional statistics in Ref. [2]—for the case of outer-edge-mode interference at $\nu = 2/5$ is ascribed to these differences by the authors.

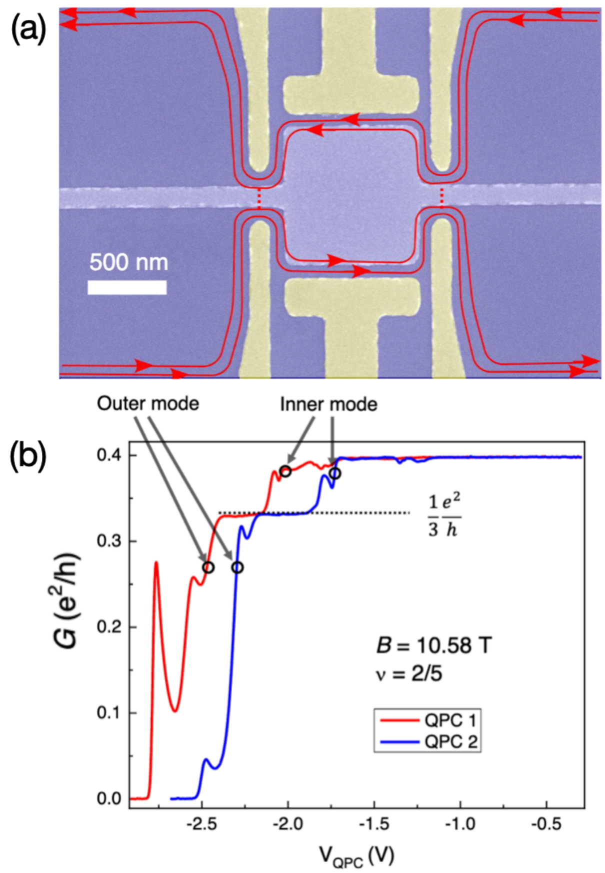

Figure 1: (a) Stylized micrograph from [20] showing a gate-defined quantum dot with point contact leads controlling edge-mode transmission and an internal gate controlling the density in the dot. The quantum dot is large, containing ~ 1000 electrons, such that the charging energy is small and localized quasipartices in the middle of the dot can be weakly coupled to the edges. (b) Depleting the two quantum point contacts allows separate controlled transmission of the inner (2/5) and outer (1/3) edge modes.

Then, moving to interferometry of the inner edge at $\nu = 2/5$, the authors deduce an effective charge $e^* \sim 0.17$, in reasonable agreement with the theoretically expected value of 1/5, based on the period of Aharonov-Bohm like oscillations along diagonals in field-gate voltage space, and more importantly, deduce a value of the statistical phase $\theta_a \sim 0.43$ consistent with the expected value of 2/5. Extracting the statistical phase involves measuring both phase jumps and a bulk-edge coupling parameter, which necessarily adds complexity and allows some room for interpretation.

The experiment demonstrates not only the value of designing heterostructures, and not just gate patterns, but also reveals the complexity of interferometry experiments, especially when more than one edge mode is involved.

Besides Ref. [20], other recent investigations of fractional charge and statistics have also shown impressive progress. An alternative approach based on a Mach-Zender interferometer [5] avoids confinement by design, and so only sees Aharonov-Bohm interference. This study of the interference pattern produced by the outer (1/3) edge at bulk filling factor $\nu = 2/5$ measures a flux periodicity of $1\Phi_0 = h/e$ while simultaneously measuring an effective charge $e^* = 1/3$ based on shot noise. The authors point out that these two results taken together reflect a fractional braiding phase of $2\pi/3$, as expected for 1/3 fractional quasiparticles. A related argument for the amplitude of the interference distinguishes electrons and 1/3 anyons with reasonable consistency between theory and experiment.

Although results from Ref. [20] (as well a Ref. [5]) appear compelling, and indeed conform to the simplest theoretical expectations to a remarkable extent, the case is certainly not yet closed. As was recently stressed in Ref. [4] and many earlier papers referenced therein, there are a variety of theoretically expected confounding factors, so that in some ways the fact that the results look so good is itself something of a mystery. There is a general expectation that quantum Hall edges are considerably more complicated than the minimal edge-modes treated in the simplest theories. Still more importantly, localized quasi-particles near the edges of the sample can be expected to change their state as a function of changing field or changing gate voltage, which (if they are close enough to the edge to tunnel from inside to outside) can lead to additional complicated changes in the interference patterns. (So, for instance, related earlier work[6] on deducing the effective charge from interferometry in the vicinity of $\nu = 5/2$ – where there is an even larger number of expected edge modes – is even more complex, leaving correspondingly more room for alternative interpretations of the data.)

Fortunately, there is still much to understand, and plenty more regimes and device geometries to explore. But, as a community, we are beginning to get a handle on the important problem of anyon braiding and statistics.

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