Journal Club for Condensed Matter Physics DOI:10.36471/JCCM_June_2023_02
https://www.condmatjclub.org

Fractionalized electrons without a magnetic field

1. **Signatures of Fractional Quantum Anomalous Hall States in Twisted $MoTe_2$ Bilayer**
   Authors: Jiaqi Cai, Eric Anderson, Chong Wang, Xiaowei Zhang, Xiaoyu Liu, William Holtzmann, Yinong Zhang, Fengren Fan, Takashi Taniguchi, Kenji Watanabe, Ying Ran, Ting Cao, Liang Fu, Di Xiao, Wang Yao, Xiaodong Xu
   arXiv:2304.08470

2. **Integer and fractional Chern insulators in twisted bilayer $MoTe_2$**
   Authors: Yihang Zeng, Zhengchao Xia, Kaifei Kang, Jiacheng Zhu, Patrick Knüppel, Chirag Vaswani, Kenji Watanabe, Takashi Taniguchi, Kin Fai Mak, Jie Shan
   arXiv:2305.00973

Recommended with a Commentary by Chris R. Laumann, Boston University

Two recent articles [1, 2] report observing signatures of the Fractional Quantum Anomalous Hall (FQAH) effect in twisted bilayer $MoTe_2$. These are the first experimental evidence of fractional quantum Hall (FQH) states in the absence of an applied magnetic field.

Let's work backwards through the acronym FQAH to unpack what precisely this claim means, notably as there are multiple terminologies. The *Hall* effect refers to the Hall conductivity, σ_{xy}, which quantifies the current j_x induced by a perpendicular electric field E_y – or more precisely, the antisymmetric part of the conductivity tensor. Classically, the Hall effect arises due to the Lorentz force on the carriers from a magnetic field B piercing the sample [3]. The Hall effect is thus *Anomalous* if σ_{xy} is non-zero in the absence of an applied B field [4]; this requires some other source of time reversal breaking. The Hall effect is *Quantum* if the Hall conductivity is quantized in fundamental units

$$\sigma_{xy} = t\frac{e^2}{h} \tag{1}$$

while the normal dissipative conductivity vanishes. Here, t is either an integer or small-denominator *Fraction* (in the FQH) [5]. Thus, a FQAH state should exhibit a fractionally quantized Hall conductivity in the absence of magnetic field.

The FQH states on lattice systems are also referred to, essentially interchangeably, as Fractional Chern Insulators (FCI) [6]. The term Chern Insulator derives from the mathematical connection between the quantum Hall conductivity in a gapped insulator and the

Chern number of the ground state on a torus [7]. This is a somewhat more modern perspective, which de-emphasizes the fractional Hall conductance relative to the other aspects of topological order which are believed to come along with it, such as quasiparticles with fractional charge and exchange statistics. It also focuses attention on the lattice rather than the magnetic field as the microscopic source of time-reversal breaking [8].

The Evidence— Given the discussion above, it might be surprising that neither group has measured the Hall conductivity of their twisted $MoTe_2$ samples, or anything about fractional quasiparticles. Rather, the essential physical principle underlying the interpretation of Refs. [1, 2] is that of incompressibility. A system of electrons is incompressible if there is a finite energy gap to add or remove an electron to the bulk – which, at least in the absence of disorder, we expect of insulating states such as the QH states. Incompressible states appear as peaks in the incompressibility as a function of the density of electrons, ρ_e. In Ref. [1], the incompressibility is measured indirectly from the shifts in the photoluminescent spectrum of a probe exciton, while in Ref. [2], a capacitively coupled sensing layer is used as a direct probe.

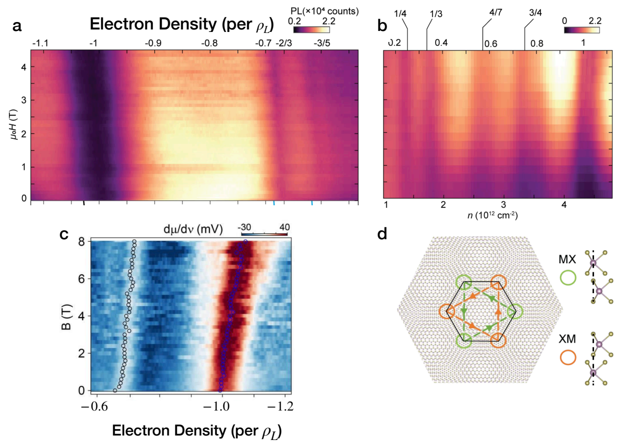

Figure 1: Incompressible states appear as peaks in the incompressibility, as measured by optical proxy in (a,b) from Ref. [1] and by capacitive sensor (c) from Ref. [2]. At zero field, on the hole-doped side, incompressible states appear at commensurate lattice fillings $s = -1, -2/3$ in (a,c) and weakly at $s = -3/5$ in (a). Note the electron density axis is reversed for panel (c) as Ref. [2] takes holes to have positive filling. The quantized Hall conductance is indicated by the linear dispersion with applied magnetic field, corresponding to $t = -1, -2/3, -3/5$ respectively. (b) This evolution contrasts with the incompressible states on the electron-doped side, whose density does not depend on B. (d) Sketch of Moiré super lattice.

Typically, in zero magnetic field, incompressible states appear at electron density commensurate with the background lattice. For example, band insulators have integer filling s with respect to the unit cell and simple charge density waves have fractional filling such as $1/4$. In the small-angle twisted bilayer $MoTe_2$ studied in both experiments, the relevant lattice is the Moiré superlattice, visible in Fig. 1d, a honeycomb lattice with large side length $a_L \approx 5 - 6$ nm and correspondingly low density $\rho_L \approx 3 - 4 \times 10^{12}$ cm^{-2}. The proposed QAH states appear as incompressible peaks on the hole doped side at fillings $s = -1, -2/3$ in both Refs. [1, 2] and additionally (weakly) at $s = -3/5$ in Ref. [1], see Fig. 1a,c.

To identify the states associated with these peaks as quantum Hall states, the authors consider how they respond to applied magnetic fields. For incompressible states, the Streda formula [9] relates the Hall conductance to the change in the electron density ρ_e as a function of applied field B,

$$\sigma_{xy} = e\frac{\partial \rho_e}{\partial B} \tag{2}$$

Or, integrating this relationship from $B = 0$ and rearranging, we find that the electron density is a rational linear combination of the density of lattice unit cells ρ_L and the density of magnetic flux quanta $\rho_B = B/(h/e)$,

$$\rho_e = s\rho_L + t\rho_B \tag{3}$$

With this relation in hand, we return to Fig. 1 and look at the evolution of the incompressible peaks in a magnetic field. These are indeed linear in B (from zero to quite large fields of 8 T in Ref. [2]) and can be labeled by their slope and intercept $(s, t) = (-1, -1), (-2/3, -2/3), (-3/5, -3/5)$. These can be contrasted with simpler incompressible states which have no evolution with B ($t = 0$) on the electron doped side – presumably various charge density waves.

Where does the time-reversal breaking at $B = 0$ come from? Both groups use optical techniques to check that the system exhibits out of plane ferromagnetism, which spontaneously breaks the time reversal symmetry required to exhibit a Hall response. Indeed, the coercive field of the ferromagnet also reflects the formation of the incompressible state and is used as another diagnostic of the FQAH states. On the other hand, one should not think of the FQAH state as arising simply as if there were a large local field induced by the ferromagnetism. In a thin film, the B field produced by a uniform magnetization vanishes in the bulk. Rather, from a mean-field perspective, the magnetic ordering presumably produces Berry curvature in the effective electronic bands and residual interactions stabilize the incompressible FQAH states at partial filling. It would be very interesting if more detailed transport experiments could observe the Berry curvature directly.

The Streda formula Eq. (3) can be proven quite generally for non-interacting systems, where s, t must take integer values. It is also intuitively clear in the traditional Landau level analysis of FQH in semiconductors in large B fields: the s electrons bound to the lattice can be thought of as the atomic core electrons while the t electrons are the carriers which populate the Landau levels – whose degeneracy is precisely given by the number of magnetic flux quanta.

The FQAH states with $s = t = -2/3$ in the zero field limit are quite a bit stranger. Here, the relevant electrons form an incompressible state with a density commensurate with the lattice unit cell. The electrons in this liquid *decouple* from that lattice unit cell as the magnetic field turns on – at 8T, the magnetic length ~ 8 nm is nearly as short as the lattice length $a_L \sim 5$ nm and yet the state evolves smoothly!

The discovery of the fractional quantum Hall effect in semiconductor heterostructures subject to large magnetic fields about four decades ago led to significant revision of how we classify and understand quantum many-body phases. The FQH states remain the prototypical topological phase, especially on the experimental front where a variety of FQH states have been studied in both semiconductor heterostructures and 2D layered materials. The measurements reported here reveal a new platform for studying such physics without the complications of large magnetic fields. There is a long and exciting road ahead to measurements of other aspects of topological order.

References

[1] J. Cai, E. Anderson, C. Wang, X. Zhang, X. Liu, W. Holtzmann, Y. Zhang, F. Fan, T. Taniguchi, K. Watanabe, Y. Ran, T. Cao, L. Fu, D. Xiao, W. Yao, and X. Xu, Signatures of Fractional Quantum Anomalous Hall States in Twisted MoTe2 Bilayer (2023), arxiv:2304.08470 [cond-mat] .

[2] Y. Zeng, Z. Xia, K. Kang, J. Zhu, P. Knüppel, C. Vaswani, K. Watanabe, T. Taniguchi, K. F. Mak, and J. Shan, Integer and fractional Chern insulators in twisted bilayer MoTe2 (2023), arxiv:2305.00973 [cond-mat] .

[3] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt-Saunders, 1976).

[4] C.-Z. Chang, C.-X. Liu, and A. H. MacDonald, Reviews of Modern Physics **95**, 011002 (2023).

[5] S. M. Girvin and K. Yang, *Modern Condensed Matter Physics*, 1st ed. (Cambridge University Press, Cambridge ; New York, NY, 2019).

[6] T. Neupert, L. Santos, C. Chamon, and C. Mudry, Physical Review Letters **106**, 236804 (2011); N. Regnault and B. Bernevig, Physical Review X **1** (2011); D. N. Sheng, Z.-C. Gu, K. Sun, and L. Sheng, Nature Communications **2**, 389 (2011).

[7] D. J. Thouless, M. Kohmoto, M. Nightingale, and M. den Nijs, Physical Review Letters **49**, 405 (1982).

[8] F. D. M. Haldane, Physical Review Letters **61**, 2015 (1988).

[9] A. Widom, Physics Letters A **90**, 474 (1982); P. Streda and L. Smrcka, Journal of Physics C: Solid State Physics **16**, L895 (1983); A. H. MacDonald and P. Středa, Physical Review B **29**, 1616 (1984).