Journal Club for Condensed Matter Physics DOI:10.36471/JCCM_July_2023_03
https://www.condmatjclub.org

Experimenting with non-abelian anyons made of qubits

1. **Non-Abelian braiding of graph vertices in a superconducting processor**
   Authors: Google Quantum AI and Collaborators
   Nature **618**, 264 (2023).

2. **Digital Simulation of Projective Non-Abelian Anyons with 68 Superconducting Qubits**
   Authors: Shibo Xu et al.
   Chinese Physics Letters, **40**, 060301, (2023).

3. **Creation of Non-Abelian Topological Order and Anyons on a Trapped-Ion Processor**
   Authors: Mohsin Iqbal et al.
   arXiv:2305.03766

Recommended with a Commentary by Ady Stern (Weizmann Institute of Science) and Steven Simon (Oxford University)

In two dimensions, topologically ordered systems have a number of extraordinary properties [1, 2]. The first is a degeneracy of the ground state when the system is placed on a topologically nontrivial surface (say a torus). Unlike more "commonplace" degeneracies, this degeneracy does not require any symmetry to be preserved, and cannot be lifted by local variations of the details of the system. Rather, the multiplicity of the degenerate ground state depends on the topology of the surface, and the degenerate ground states are separated from the rest of the spectrum by an energy gap. Second, topologically ordered systems have excitations which are anyons — that is, quasi-particles that are neither bosons nor fermions. Anyons may be"abelian," in which case a clockwise adiabatic braiding of one anyon around the other applies a topological phase $e^{i\theta}$ to the ground state wavefunction, and counterclockwise braiding applies the inverse phase. But they may also be non-abelian. In this case, the degeneracy of the ground state increases exponentially as more anyon excitations are added to the system at some fixed positions. Then, braiding anyons around one another does not merely multiply the ground state by a phase. Rather, it implements a unitary operation that transforms the initial ground state to another ground state. Unitary transformations do not generally commute, hence the name non-abelian. Importantly, as long as the braiding is carried out adiabatically with respect to the energy gap, the unitary transformation associated with the braiding does not depend on the geometry of the trajectory carrying the braiding out, but on its topology only.

Further to the great interest anyonic states raise due to their unique properties, non-abelian anyons also have a potential use for quantum information processing. Qubits can be stored in the Hilbert space spanned by the degenerate ground states, and can be manipulated by braiding the anyons around each other — a scheme known as "topological quantum computation" (TQC) [2].

Although the notions of anyons and topological order are approaching their half-century anniversary, the number of examples we have definitively found in experiment is somewhat limited. The major line of exploration has so far been the fractional quantum Hall effect (FQHE), whose mere existence requires the existence of anyons. Some beautiful recent experiments on fractional quantum Hall systems have observed abelian anyonic effects on electronic transport. These experiments[3], which were described in several recent Journal Club articles[4], involved some 10^8 electrons whose Hamiltonian dictated their low energy dynamics to be well described in terms of anyons.

A new approach to study anyons, which is the subject of this journal club, is fundamentally different. In this approach the system is small — a few dozen qubits — in a present day quantum computer. The dynamics are not dictated by the low energy eigenstates of a time independent Hamiltonian, but rather by detailed driving applied to the qubits by the experimentalists. The first goal of this driving is to transform a simple initial state of the collection of qubits — say a product state — to an "anyonic wave function" of the qubits. This wave function is characterized by long-ranged quantum entanglement, and its creation requires a well designed set of quantum gates, possibly interspersed with measurements. Furthermore, the wave function hosts anyonic localized topological defects, whose braiding the experiment is designed to demonstrate. After the wave-function is created, it is manipulated by another set of quantum gates that carry out the braiding, until the final state is examined. This new approach is well suited for small scale systems — it would have been utterly impossible to control the wave function of some millions of electrons in a two-dimensional conductor.

The new approach is profoundly different from the traditional electronic-based approach. The wave function that is generated is not the ground state of any Hamiltonian. Its generation and manipulation are not protected by any energy gap. Anyons in this approach are not excitations. A new intuition needs to be developed. It is useful to be inspired by the language of quantum error correction to distinguish the new approach from the old one. Rather than counting on adiabaticity and energy gaps to keep the system in its ground state subspace, we now rely on repeated error-check (or "syndrome") measurements. Such measurements tell the experimentalist if the system has incorrectly evolved to a state which is out of the set of states it should be limited to (the "code space", which is the analog of the ground states subspace). When such an error is detected, the experimentalist can actively apply gates to remove these errors and push the system back to the desired subspace. For systems with non-abelian anyons, the error-check operators indicate the presence of anyons. When needed, for example, when stray pairs of anyons are generated from the vacuum, errors are removed by pushing anyons together and annihilating them. When the measurements are frequent enough, they protect the braiding process, and guarantee its topological outcome.

Several experiments have recently been carried out within this new approach, albeit with a limited application of error detection and correction. The first among these experiments demonstrated braiding of abelian anyons in the toric (surface) code [5]. This state ("code") carries two types of anyons, commonly referred to as "electric" and "magnetic." When an anyon of one type encircles an anyon of the other type, an anyonic phase is accumulated. This phase may be encoded on an ancilla qubit, and then be read out.

The three recent experiments that are listed above have now extended the above approach to demonstrate non-abelian braiding. Two of these experiments, by a Google team and its collaborators and by Xu et al., address similar non-abelian settings, using the platform of superconducting qubits. The starting point of these experiments is the abelian toric code. The experiments are based on the theoretical observation[1] that defects in the lattice on which the system resides are endowed with three remarkable properties. First, when an electric anyon encircles such a defect, it is transformed to being a magnetic anyon, and vice versa. Second, the defects enlarge the Hilbert space of states characterized by the "correct" eigenvalues of the projection operators. This is the analog of the enlargement of the ground state degeneracy when anyons are added to a system whose Hamiltonian describes a non-abelian topological order. Third, when two defects are brought together, they may "fuse" in more than one way, distinguished by the presence or absence of a local fermionic excitation. And fourth, as pointed out theoretically in Ref. [6], when the position of the defects is braided, the state of the system is unitarily transformed between these "ground states." The combination of these properties endows these objects with the name "Projective anyons." The two experiments linked above create these projective anyons in the lab, and beautifully demonstrate all their defining properties. Furthermore, the experiments have also demonstrated the use of the set of possible fusion channels of the projective anyons as logical qubits.

In contrast, the experiment by Iqbal et al. uses a trapped ion quantum processor in a race-track geometry, and places bits on a simulated torus surface. The ability to have long-range connections, which are the characteristic point of strength of trapped ions systems, allows the connection of the bits into the simulated torus, thus providing a way to measure the analog of the ground state degeneracy on the torus. On that torus, this experiment constructs and studies a novel type of non-abelian topological order, and its anyons. The non-abelian state is produced by starting with three copies of a toric code lattice. The key step that turns the abelian toric code into a non-abelian topological order involves an intricate entanglement of these three copies. Importantly, were the resulting state produced in a Hamiltonian system, the non-abelian anyons would have been dynamical excitations, as opposed to the defect-bound "projective" anyons we described before. The experiment demonstrated non-abelian features of braiding as well as the correct "ground state degeneracy" of the expected non-abelian order on this toroidal surface.

Despite the impressive properties of present day quantum computers, after some number of operations, errors build up and the wave-function is overtaken by noise. The sophisticated schemes of error detection and correction are beyond the reach of existing experiments. The effect of noise is particularly problematic being that the number of unitary operations necessary to entangle qubits into a topologically ordered state grows with the size of the system. To skirt around this limitation, Iqbal et al. used a so-called "measure-and-feed-forward" protocol, which is a partial error-correction scheme. This protocol invokes measurements (a nonunitary operation) and then applies unitary time evolution that depends on the result of the measurement. This enables the creation of topological order in a time which does not grow with the system size.

The experiments that we present here are formidable preliminary steps — testing some of the fundamental notions of non-abelian anyons that have been around for many years without any detailed experiments. Still the transition from these experiments into quantum information processing requires overcoming some major challenges. First, error correction will need to be implemented in a fuller manner to obtain real topological protection of quantum information, in particular at the required larger scales of qubit numbers and over longer times. Second, even if fully error corrected, none of the topological orders that have been demonstrated so far is "universal," i.e., sufficient for the realization of a set of gates that can carry out all possible unitary transformations by braiding and measuring. Nonetheless, even at this stage, these experiments present remarkable achievements and pose exciting challenges for theorists and experimentalists alike. These challenges are somewhat of opposite nature. While experimentalists will put their effort on removing the limitations of present day devices, theorists will search for ways of implementing more and more types of topological order while living within the inevitable limitations that will remain.

References

[1] A. Kitaev, Annals of Physics, 321, 2, 2006.

[2] C. Nayak, S. H. Simon, Ady Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. 80, 1083 (2008).

[3] J. Nakamura, , S. Liang, S., G.C. Gardner, and M. J. Manfra, Nat. Phys. 16, 931–936 (2020); Nat Commun 13, 344 (2022); arXiv:2304.12415; H. Bartolomei et al, Science 368, 173–177 (2020), M. Ruelle et al, arXiv:2210.01066.

[4] Journal clubs: `JCCM_July_2020_01, JCCM_July_2020_02, JCCM_June_2023_01`

[5] Satzinger, K. et al. Science 374, 1237–1241 (2021); Song et al., Phys. Rev. Lett. 121, 030502; Y. P. Zhong, et al., Phys. Rev. Lett. 117, 110501 (2016).

[6] H. Bombin, Phys. Rev. Lett. 105, 030403 (2010).