Journal Club for Condensed Matter Physics DOI:10.36471/JCCM_September_2023_01
https://www.condmatjclub.org

Towards flat band superconductivity

Evidence for Dirac flat band superconductivity enabled by quantum geometry
Authors: Haidong Tian, Xueshi Gao, Yuxin Zhang, Shi Che, Tianyi Xu, Patrick Cheung, Kenji Watanabe, Takashi Taniguchi, Mohit Randeria, Fan Zhang, Chun Ning Lau, and Marc W. Bockrath
Nature **614**, 440 (2023) — arXiv:2112.13401

Recommended with a Commentary by Carlo Beenakker,
Instituut-Lorentz, Leiden University

The London equation is a linear relation $\boldsymbol{j} = -D^s\boldsymbol{A}$ in a superconductor between the electrical current density $\boldsymbol{j}$ and the vector potential $\boldsymbol{A}$ in the gauge div $\boldsymbol{A} = 0$. The proportionality coefficient D^s is called the superfluid weight. At zero temperature, for a parabolic band with effective mass $m_{\rm eff}$, the superfluid weight equals $D^s_0 = e^2\tilde{n}/m_{\rm eff}$, with $\tilde{n}$ the number density of conduction electrons. The superfluid weight governs the electromagnetic properties of superconductors, notably the Meissner effect.

In a two-dimensional (2D) system, the product $(\hbar/e)^2 D^s$ has dimension of energy. For 2D free electrons this is the Fermi energy $E_{\rm F} = \hbar^2 k_{\rm F}^2/2m_{\rm eff} = \pi(\hbar/e)^2 D^s_0$. While the superconducting gap Δ_0 sets the temperature scale at which D^s vanishes, the low temperature value of the superfluid weight is independent of the pairing interaction.

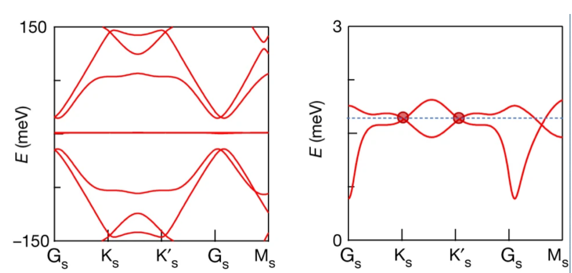

Fig. 1. Calculated band structure of twisted bilayer graphene at $\theta = 1.08°$. The right panel zooms in on the flat band, the red dots indicate the two Dirac points. (From Tian *et al.*)

All of this assumes that $\Delta_0 \ll E_{\rm F}$. In the flat-band limit $m_{\rm eff} \to \infty$ the Fermi energy vanishes, but the superfluid weight may retain a nonzero value, set by the pairing interaction. What is needed is that the energy band is flat (dispersionless) because of destructive interference of wave functions extended over many sites, rather than because of on-site localization. The vector potential changes the interference pattern, allowing for a nonzero supercurrent. An integer invariant of the flat band, the Chern number, provides a lower bound for the superfluid weight — a remarkable finding by Peotta and Törmä. Because the Chern number is a geometric property of quantum states one says that "flat band superconductivity is enabled by quantum geometry". The paper by Tian *et al.* reports evidence for this effect. Let me walk you through it.

The 2D flat band material is a pair of graphene monolayers, stacked with a small angular mismatch θ. The low-energy excitations of this twisted graphene bilayer are massless Dirac fermions, as in the monolayer, but the Fermi velocity is much smaller. A dispersionless band appears near a "magic" twist angle around 1°. The band structure at $\theta = 1.08°$ is shown in Fig. 1. The flat band near the Fermi energy has a width of about 1 meV. In the experiment an average $\bar{v}$ of the carrier velocity over the flat band is measured from the nonlinear current-voltage characteristic in the normal state: the differential resistance dV/dI has a peak when $I = e\bar{v}\tilde{n}$. The resulting $\bar{v} \approx 10^3$ m/s is close to the slope of the dispersion at the Dirac points in Fig. 1. Note that this value is one thousand times as small as in the graphene monolayer.

As discovered by Jarillo-Herrero and his group, magic-angle twisted bilayer graphene becomes superconducting at temperatures below 2 K (see the April 2018 Journal Club issue). The critical supercurrent j_c is reached at $A = (\hbar/2e)\xi^{-1}$. The superconducting coherent length ξ is related to the upper critical magnetic field B_{c2} via $B_{c2}\xi^2 = \hbar/2e$. Measurements of j_c and B_{c2} thus allow determination of the superfluid weight,

$$D^s = \frac{2e}{\hbar}\xi j_c = \sqrt{\frac{2e}{\hbar B_{c2}}} j_c. \tag{1}$$

The resulting density dependence of D^s is shown in Fig. 2 (red curve). The black and green curves would result, respectively, by identifying $(\hbar/e)^2 D^s$ with the Fermi energy $E_{\rm F}$ or the superconducting gap Δ_0. It is evident from the comparison that the superfluid weight of the twisted graphene bilayer is governed not by the kinetic energy but by the pairing interaction. How does this support the claim of "superconductivity enabled by quantum geometry"?

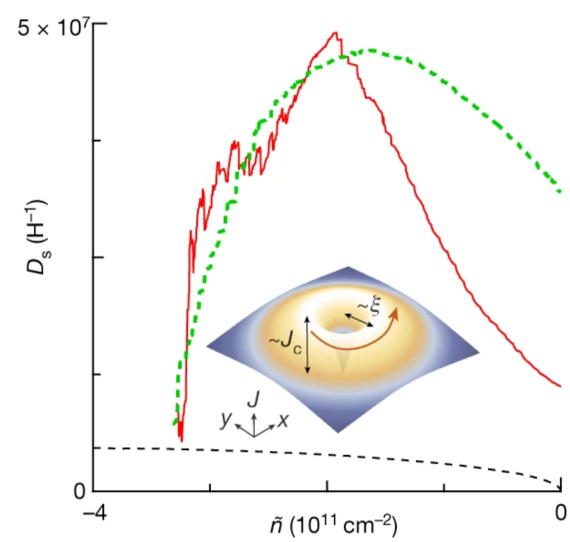

Fig. 2. Red curve: Density dependence of the superfluid weight, calculated using Eq. (1) from the measured critical current density and upper critical magnetic field. The black line is the London value $D^s_0 = e^2\tilde{n}/m_{\rm eff}$, with $m_{\rm eff} = (\hbar/\bar{v})\sqrt{2\pi\tilde{n}}$. The green curve is the estimate $D^s = c(e/\hbar)^2\Delta_0$, with fit parameter $c = 0.33$.(From Tian *et al.*)

In the theory of flat band superconductivity (reviewed recently by Törmä, Peotta, and Bernevig) one distinguishes intraband from interband contributions to the supercurrent. The distinction refers to diagonal and off-diagonal matrix elements of the velocity operator $\boldsymbol{v} = \partial H/\partial \boldsymbol{k}$ in the basis of single-particle Bloch states,

$$H(\boldsymbol{k})|m,\boldsymbol{k}\rangle = \varepsilon_m(\boldsymbol{k})|m,\boldsymbol{k}\rangle \Rightarrow \tag{2}$$
$$\langle m|\boldsymbol{v}|n\rangle = \delta_{nm}\partial_{\boldsymbol{k}}\varepsilon_n + (\varepsilon_n - \varepsilon_m)\langle m|\partial_{\boldsymbol{k}}|n\rangle .$$

The interband ($n \neq m$) contributions are called "geometric" because $\langle m|\partial_{\boldsymbol{k}}|n\rangle$ can be used to quantify the distance of two eigenstates in the Brillouin zone.

In a completely dispersionless band there are no conventional, intraband contributions to the superfluid weight. Only the geometric, interband contributions remain. In the experiment the superconducting gap Δ_0 in the range 0.1–0.4 meV is smaller than the band width $W \approx 1$ meV, so both classes of contributions are present. Still, a calculation by Hu, Hyart, Pikulin, and Rossi (see Fig. 3) indicates that the geometric contribution dominates when Δ_0 exceeds 10% of the band width, which applies to the experiment.

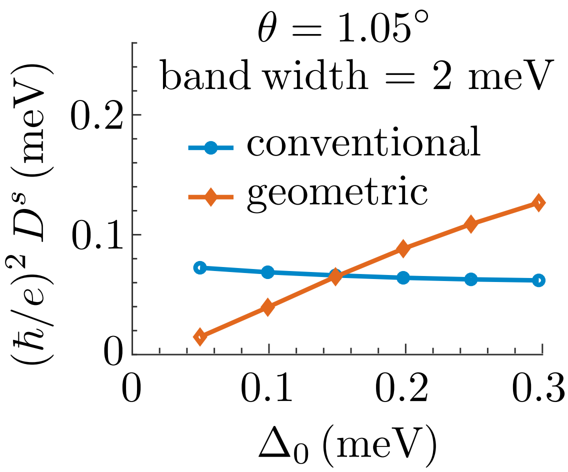

Fig. 3. Dependence of the superfluid weight on the gap Δ_0, calculated for twisted bilayer graphene at $\theta = 1.05°$. The conventional contribution is insensitive to Δ_0, while the geometric contribution increases with increasing Δ_0. (From Hu *et al.*)