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Absolutely stable local quantum memories in physically accessible dimensions

A local automaton for the 2D toric code

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Recommended with a Commentary by Rahul Nandkishore,

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What are the possible phases of non-equilibrium matter? This, to my mind, is one of the great open questions for theoretical condensed matter physics. Notably, in contrast to systems in equilibrium, there does not even exist a consensus definition of how one should define a 'phase of matter' in the non-equilibrium setting. Nonetheless, I will argue that the recommended paper presents a construction that should be considered a new kind of non-equilibrium phase of matter, under any reasonable definition of the term.

There are multiple useful axes along which nonequilibrium many body systems may be categorized. We could be discussing systems that are classical or quantum. We could be discussing systems with 'passive' dynamics (i.e. a time independent Hamiltonian), or with 'active' dynamics (incorporating e.g. measurements and feedback). We could be discussing systems where the dynamics is *local* (in space and/or time) or not. And we could be discussing systems that can be embedded into a physically reasonable number of spatial dimensions ($d = 0, 1, 2, 3$), or not. The recommended paper considers dynamics which is *quantum*, *active*, *local* and in $d = 2, 3$.

The 'intellectual parentage' of the recommended work is two-fold. On the one hand, it may be viewed as a new entry to the broad field of *quantum error correcting (QEC) codes* i.e. systems into which quantum information may be encoded, in a manner that is robust to imperfections and noise. It differs crucially, however, from standard QEC codes in $d \leq 3$ in that the error correction only involves feedback that is local in space and time. On the other hand, it may be viewed as an extension of certain results in *classical* cellular automata (notably the Toom and Gacs automata), to the quantum regime. It is the latter perspective from which I will approach the authors result.

Suppose one wished to robustly encode a single bit of *classical* information into a classical many body system. The simplest solution would be to take the classical many body system to be an Ising ferromagnet, the magnetization of which corresponds to the classical bit. As long as there are two degenerate 'equilibrium' states, the Ising ferromagnet can function as a 'classical memory.' For a system in $d = 2$, this 'classical memory' will even survive weak thermal noise (because the Ising ferromagnet survives to non-zero temperature), but it will not survive application of a longitudinal magnetic field (equivalently, biased noise), since any magnetic field will lead to there being only one equilibrium state. Thus, the Ising ferromagnet can serve as a good classical memory only if we impose a global Ising symmetry. If we upgrade from passive to active (but still local) dynamics, we can get a classical memory that survives weak longitudinal field (biased noise), as shown by Toom. The basic idea behind Toom's protocol is to still encode the classical bit into the global magnetization, and to use a version of 'local majority vote' to correct errors introduced by noise. Now, the problem with a simple 'majority vote' protocol is that if you get domains in your system, then majority vote across the domain wall is inconclusive. Toom solves this problem using a simple geometric observation - namely that the *minority* domain must have convex corners. Accordingly, any time you encounter a convex corner you can just shrink the domain that has the convex corner, and iterating this process eliminates minority domains. The resulting active dynamics has two possible steady states even in the presence of weak biased noise, and can thus serve as a good classical memory, robust to arbitary noise (without symmetry restriction)

The (hard to understand but believed to be true) results of Gacs (see also GacsII) can be understood as a massive generalization of the Toom automaton to *one* spatial dimension. They consider a *one dimensional* local active dynamics, which preserves an *extensive* number of bits of information in local observables for infinite times, robust to weak but arbitrary noise. This is particularly striking since under *passive* local dynamics, one dimensional systems cannot even preserve a single bit of information in the presence of noise, even with imposed symmetries - the one dimensional Ising model has only one possible equilibrium state at non-zero temperature. Nonetheless, the Gacs automaton (active local dynamics in one dimension) has *O(L)* possible steady states, which differ in their observable properties in a manner robust to arbitrary weak noise. Whatever definition one prefers for phases of classical non-equilibrium matter, the Gacs automaton should certainly count.

Can we have a *quantum* generalization of the Toom and Gacs automata? That is, can we have a quantum active dynamics, in a physically reasonable number of dimensions, into which we can encode *quantum* information (both bit and phase) in a manner robust to arbitrary weak noise? And can we do it in a way that does not require feedback that is nonlocal in space and time? The recommended paper answers this question in the affirmative. To do so, it leans on a previously obscure classical construction named the Tsirelson automaton. The Tsirelson automaton works in one spatial dimension, and starts in the same place as the above mentioned Toom automaton - by encoding the classical information into the magnetization of an Ising ferromagnet, and correcting errors introduced by noise using a 'majority vote' type of procedure. However, it deals with the problem of minority domains differently to Toom. Namely, it deals with minority domains using a 'divide and conquer' strategy. First, one blocks the system into spatially contiguous regions on which one applies majority vote. Then, each 'block' is carved up into smaller pieces, with are redistributed through the system (see Fig.1). This process is iterated. Eventually, any minority domain gets chopped up and redistributed into small enough pieces, to be correctable via local majority vote.

In the recommended paper, Balasubramanian, Davydova and Lake take inspiration from the Tsirelson classical automaton (above) to design an active quantum dynamics that can preserve *quantum* information using only feedback that is local in space and time. Their construction is based on the well known 'two dimensional toric code.' The two dimensional toric code has point-like excitations, the proliferation of which can cause logical errors. This problem can famously be 'solved' by going to four spatial dimensions, whereupon the pointlike excitations get replaced by loops, which need to grow to be of order system size in order to cause logical errors. The recommended paper shows how the problem can also be solved in physically reasonable dimensions, by replacing passive by active (still local) dynamics.

Two versions are presented. First a construction is presented in strictly two dimensions. The dynamics in this case is 'hard wired' and not translationally invariant in time - an initial state

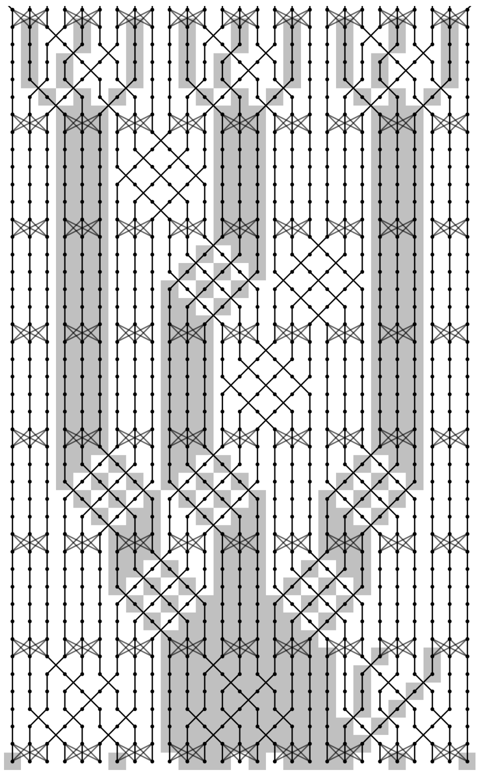

Figure 1: Figure taken from the recommended paper, illustrating the circuit realizing Tsirelson?s automaton. The time direction points up. The minority domain corresponds to cells shaded in gray. Here, information is encoded into the magnetization of a one dimensional Ising system. The dynamics involves a sequential ‘majority vote’ on local regions, followed by a carving up and redistribution of said regions. Iterating this process produces an active dynamics that is capable of correcting minority domains, and thus can encode information robustly to weak noise without demanding any symmetry.

corresponding to a two dimenisonal toric code is taken, and then a particular active local dynamics is applied, which is shown to be capable of preserving two qubits of quantum information (on a torus), robust to arbitrary noise. The necessary operations are allowed to be faulty (as long as the error rates are low enough). Next, an extension to three dimensions is presented, which can preserve L qubits of information (L being in the number of layers), and with a dynamics which is translation invariant in time (but not space). Essentially, the information is shuttled between layers in such a way as to reproduce the (non-time-translation-invariant) dynamics in the pure-two dimensional construction. The constructions presented here constitute examples of *absolutely stable* quantum memories, robust to arbitrary noise, and employing only feedback that is local in both space and time, in a physically reasonable number of dimensions ($d \leq 3$). I believe they should be considered a new type of non-equilibrium quantum phase of (active) matter. It seems clear that this construction should be extendable to other topologically ordered phases, and probably also to fracton phases. Whether any surprises are encountered in the process remains to be determined. More broadly, the recommended manuscript constitutes a new front in the exploration of 'interactively operated' many body quantum systems, this time with the crucial addition of geometric locality. It seems likely that there is much more to be done in this direction.