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Can the $\nu = 1/3$ Laughlin liquid freeze?

Instability of Laughlin FQH liquids into gapless power-law correlated states with continuous exponents in ideal Chern bands: rigorous results from plasma mapping
Authors: Saranyo Moitra and Inti Sodemann Villadiego
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Recommended with a Commentary by Trithep Devakul, *Stanford University*

The celebrated Laughlin [1] wavefunction in a magnetic field B ($e = \hbar = 1$)

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) = \prod_{i<j}(z_i - z_j)^m \prod_i e^{-\frac{1}{4}B|\boldsymbol{r}_i|^2}, \qquad z_i \equiv x_i + iy_i \tag{1}$$

describes incompressible fractional quantum Hall fluids at filling fraction $\nu = 1/m$ of the lowest Landau level. Yet, as Laughlin himself noted, for sufficiently low electron densities ($\nu \lesssim 1/70$), this same wavefunction ceases to describe a liquid and instead describes a *crystalline* state. This crystallization is not the familiar Wigner crystal of a dilute electron gas, but rather an intrinsic property of Eq 1. In practice, this critical density is so small as to be irrelevant, and the Laughlin state is often regarded as synonymous with the fractional quantum Hall effect.

The highlighted paper points out that this critical density can, under certain conditions, be dramatically enhanced — so much so that even the beloved $\nu = 1/3$ Laughlin state could freeze. The resulting state describes not a Wigner crystal, but something much stranger. Such conditions may be relevant to two-dimensional material platforms where flat Chern bands are realized.

The plasma analogy. To understand why Eq 1 might freeze, the wavefunction amplitude $|\Psi(\{\boldsymbol{r}_i\})|^2 \propto e^{-\beta U(\{\boldsymbol{r}_i\})}$ can be interpreted as the Boltzmann weight of a classical 2D system of N particles at inverse temperature $\beta = 2m$, with a potential

$$U(\{\boldsymbol{r}_i\}) = -\sum_{i<j} \log|\boldsymbol{r}_i - \boldsymbol{r}_j| + \frac{B}{4m}\sum_i |\boldsymbol{r}_i|^2 \tag{2}$$

which describes N particles of charge $Q = -1$, interacting via a Coulomb interaction (logarithmic in 2D), on top of a uniform background of charge density $\rho = B/(2\pi m)$. This classical model, known as the one-component plasma, has been studied in great detail: it forms a fluid at high temperatures, but freezes to a hexagonal crystal when the dimensionless parameter $\beta Q^2 = 2m$ exceeds a critical value ~ 140. Since all correlation functions computable from $|\Psi|^2$ coincide with those of the plasma, the Laughlin state itself must describe a crystal when $m \gtrsim 70$.

Aharonov-Casher bands. Rather than conventional Landau levels, the highlighted paper examines Aharonov-Casher, or "ideal", Chern bands. As first shown by Aharonov and Casher [2], Dirac electrons in a spatially inhomogeneous magnetic field $B(\boldsymbol{r})$ always possess an exactly flat zero-energy band. There is an intuitive picture for this: since a Dirac fermion in a uniform magnetic field has an exact $n = 0$ Landau level that is pinned at exactly $E = 0$ for all B, it makes sense that $E = 0$ modes persist even in an inhomogeneous $B(\boldsymbol{r})$. These bands can be understood as generalized lowest Landau levels: they share much of the same structure and also admit analytic Laughlin states. The analog of the Laughlin state in an Aharonov-Casher band is simply

$$\Psi(\{\boldsymbol{r}_i\}) = \prod_{i<j}(z_i - z_j)^m \prod_i e^{-\phi(\boldsymbol{r}_i)} \tag{3}$$

where $\phi(\boldsymbol{r})$ satisfies $\nabla^2\phi(\boldsymbol{r}) = B(\boldsymbol{r})$. For the case of a uniform field $B(\boldsymbol{r}) = B$, the choice $\phi(\boldsymbol{r}) = \frac{B}{4}|\boldsymbol{r}|^2$ reproduces Eq 1.

For a spatially periodic magnetic field $B(\boldsymbol{r}) = B + \delta B(\boldsymbol{r})$, where $B > 0$ is a uniform component and $\delta B(\boldsymbol{r})$ averages to zero, we can write $\phi(\boldsymbol{r}) = \frac{B}{4}|\boldsymbol{r}|^2 + \delta\phi(\boldsymbol{r})$, with $\delta\phi(\boldsymbol{r})$ being a periodic function of position. Applying the plasma analogy to Eq 3 results in

$$U(\{\boldsymbol{r}_i\}) = -\sum_{i<j} \log|\boldsymbol{r}_i - \boldsymbol{r}_j| + \frac{B}{4m}\sum_i |\boldsymbol{r}_i|^2 + \sum_i \frac{\delta\phi(\boldsymbol{r}_i)}{m} \tag{4}$$

which is describes the same plasma as before, but in the presence of a periodic scalar potential $\delta\phi(\boldsymbol{r})/m$. The periodic potential can be understood as arising from a non-uniformity in the background charge density, $\rho(\boldsymbol{r}) = \rho + \delta B(\boldsymbol{r})/(2\pi m)$.

Adding a periodic potential will affect the freezing transition of the plasma. When the $\delta\phi(\boldsymbol{r})$ is commensurate with the crystal, it acts as a pinning field that favors the crystalline order, increasing the critical temperature. It stands to reason, then, that the freezing transition of the Laughlin state can occur at much higher filling fractions compared to the uniform case. This is precisely the observation made in the highlighted paper.

Interest in Aharonov-Casher bands has revived in recent years due to their connection to (zero magnetic field) Chern bands in moiré systems. They appear to be ubiquitous in idealized limits of moiré continuum models, including twisted graphene, twisted $MoTe_2$, and also rhombohedral graphene. In all these systems, an effective magnetic field emerges with one flux quantum per moiré unit cell. These emergent magnetic fields can be extremely inhomogeneous.

The paper. The highlighted paper considers a somewhat simpler setup. Rather than having 1 flux quantum and $1/m$ particle per unit cell, as in the case of real moiré models, they consider a unit cell that contains m flux quanta and 1 particle. Technically, this means choosing a magnetic field configuration $B(\boldsymbol{r})$ that is different for each m. This leads to a simpler picture in the plasma analogy since the number of particles per unit cell is m-independent, and from which the authors can leverage prior studies of 2D classical plasma models.

Specifically, they consider $B(\boldsymbol{r})$ such that the plasma model describes particles moving on top of an array of classical "Thomson atoms" — uniform discs of constant charge density (Fig. 1) also known as "plum pudding" atoms. Since there is one particle per unit cell, the plasma in this case does not actually crystallize in the sense discussed earlier. Nevertheless, there is still a phase transition from a fluid (plasma) at high temperatures to a "dielectric" as temperature is decreased. Here, dielectric refers to a state in which each particle is bound to its own Thomson atom: it behaves as a conventional dielectric material. The critical temperature for this transition depends on the ratio of the radius of the Thomson atoms, σ, to their interatomic spacing, a. As the atoms become more point-like ($\sigma/a \to 0$), the critical temperature can increase up to the point that even the $\nu = 1/3$ Laughlin state sits in the dielectric phase! This happens already for $\sigma/a \approx 0.5$, when the Thomson atoms are just barely touching.

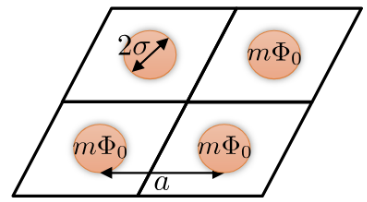

Figure 1: Magnetic field configuration, corresponding to a plasma model in an array of 2D Thomson atoms.

What happens to the Laughlin state when, according to the plasma analogy, it is a dielectric? Well, one thing is for certain: it does not describe the same fractional quantum Hall effect. The signature of the $\nu = 1/m$ fractional quantum Hall state is the existence of excitations with fractional charge e/m. To show this, Laughlin used the "perfect screening" condition of the plasma. A dielectric, however, does not fully screen! If ϵ is the dielectric constant of the classical Thomson atom array, then the effective charge of a Laughlin quasihole is

$$q_{qh} = (1 - 1/\epsilon)\frac{e}{m} \tag{5}$$

which need not be a rational fraction of e.

As the highlighted paper shows, the Laughlin dielectric exhibits power-law correlations with an exponent that depends continuously on σ/a. The power-law correlations implies that any Hamiltonian for which Eq 3 is the exact zero-energy ground state (which can be constructed using pseudopotentials) must be gapless. Since no continuous symmetry is broken in this case, this gaplessness does not appear to be a simple consequence of Goldstone theorem.

Final thoughts. To what degree is this physics relevant to real moiré systems? These systems tend to have very inhomogeneous effective magnetic fields, which is reflected in their real-space charge density (twisted bilayer graphene, for instance, has charge density concentrated at the AA sites). In reality, though, these Chern bands are not perfect Aharonov-Casher bands and, even if they were, the Coulomb ground state is not exactly the Laughlin state.

We often think of the Laughlin state as the solvable "fixed point" of a larger fractional quantum Hall phase. In an Aharonov-Casher band, can the fractional quantum Hall phase exist without a Laughlin fixed point?

References

[1] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[2] Y. Aharonov and A. Casher, Phys. Rev. A **19**, 2461 (1979).