Quasiparticle charge in the $\nu = 5/2$ fractional quantum Hall state

"Towards identification of a non-Abelian state: observation of a quarter of electron charge at $\nu = 5/2$ quantum Hall state,"
http://arXiv.org/0802.0930
Authors: M. Dolev, M. Heiblum, V. Umansky, A. Stern, D. Mahalu

"Quasiparticle Tunneling in the Fractional Quantum Hall State at $\nu = 5/2$,"
http://arXiv.org/0803.3530
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Recommended with a commentary by Kirill Shtengel, UC Riverside

Fractionalized phases are fascinating. As the name suggests, the excitations in such systems carry fractions of "normal" particles' charge and have other unusual properties, such as fractional exchange statistics. Such phases have been discovered at some fractional fillings of Landau levels of two-dimensional electron gases (2DEG) in magnetic fields. The Hall conductance σ_{xy} develops plateaux (FQH) as a function of the applied magnetic field for a fixed charge density, where it is quantized to extreme precision in fractional multiples of the conductance quantum e^2/h. These multiples are the filling fractions $\nu \equiv N_{\rm e}/N_\phi$ where $N_{\rm e}$ is the number of electrons and N_ϕ is the number of flux quanta $\Phi_0 = hc/e$ through the area occupied by the 2DEG at magnetic field corresponding to the center of a plateau. At the plateaux, the conductance tensor is off-diagonal, meaning a dissipationless transverse current flows in response to an applied electric field. In particular, the circular electric fields generated by threading an additional localized flux quantum through the system create a radial "outward" current which expels a net charge of νe, thereby creating a quasihole. Consequently, charge and flux are intimately coupled in the quantum Hall effect.

While almost all observed FQH plateaux occur at the filling fractions $\nu = p/q$ with odd denominators q, one notable exception is $\nu = 5/2$ first reported in (Willett et al., 1987). Let us ignore two completely filled lowest Landau levels (which are presumed to be inert) and concentrate on the half-filled level. The above argument for expelled charge would then suggest that the charge of a quasihole is $e/2$. However, the $\nu = 5/2$ state is not a typical FQH state. Notice that a simple Laughlin state for $1/m$ filling

$$\Psi_{\rm L} = \prod_{j<k} (z_j - z_k)^m \prod_j e^{-|z_j|^2/4} \tag{1}$$

(where $z = (x + iy)/l_0$) requires m to be odd since the manybody wavefunction for fermions has to be antisymmetric. The hierarchical descendents (i.e., Laughlin states of excitations in the parent states) also require filling fractions with odd denominators and hence cannot possibly include a $\nu = 1/2$ state. Moore and Read proposed a so-called Pfaffian trial wavefunction (Moore and Read, 1991):

$$\Psi_{\rm MR} = \mathcal{A}\left(\frac{1}{z_1 - z_2}\frac{1}{z_3 - z_4}\dots\right)\prod_{j<k}(z_j - z_k)^2 \prod_j e^{-|z_j|^2/4} \tag{2}$$

with $\mathcal{A}(\dots)$ denoting the antisymmetrized sum over all possible pairings of electron coordinates. This wavefunction has an interesting feature: within each pair (e.g. 1 & 2, 3 & 4 etc. in the term spelt out above), the electrons do not avoid their partners as much as they avoid other electrons. In fact, this wavefunction describes a weakly paired, spin-polarized superconducting state of electrons (or, more precisely, composite fermions consisting of an electron and two quanta of statistical flux).

Another possible trial state for $\nu = 1/2$ is the (3,3,1) state (Halperin, 1983):

$$\Psi_{331} = \prod_{j<k}(z_j - z_k)^3 \prod_{j<k}(w_j - w_k)^3 \prod_{j<k}(z_j - w_k) \prod_j e^{-(|z_j|^2 + |w_j|^2)/4} \tag{3}$$

which can also be interpreted as an $S_z = 0$ paired state of spin-up electrons at z_j's with spin-down electrons at w_k's. There are several other possibilities, but they all describe *paired* superconducting states. These scenarios would have a striking physical consequence: a superconducting flux quantum is $hc/2e$ – a half of the fundamental flux quantum Φ_0 – and therefore the charge it expels is half of what we argued. The charge of a quasihole should therefore be $e/4$ instead of $e/2$, providing an unmistakable signature of a paired state!

One of the oldest proposals for measuring the quasiparticle charge in the FQHE regime relies on the shot noise measurements in a quantum point contact (QPC). The idea is simple: imagine that the tunneling events are independent and hence their probability obeys a Poisson distribution $P_n = e^{-\alpha}\alpha^n/n!$. It is easy to check that the first and second cummulants are related: $\langle I\rangle\Delta t = e^*\alpha$ while $\langle (I - \langle I\rangle)^2\rangle\Delta t^2 = {e^*}^2\alpha$ and therefore $e^* = S_I(\omega = 0)/2I$. ($S_I(\omega)$ is the power spectrum of noise – the Fourier transform of the current autocorrelation function.) Independence of the tunnelling events is a good approximation in the weak tunnelling regime, and this method has been successfully used in the past to measure the fractional charge of quaisiparticles in the $\nu = 1/3$ and $\nu = 2/5$ FQH regimes (De Picciotto et al., 1997; Reznikov et al., 1999). The first of the recommended papers (by M. Dolev et al.) reports the results of the shot noise measurements in the $\nu = 5/2$ state that are consistent with the $e/4$ charge of a quasiparticle and clearly exclude the possibility of $e^* = e/2$. The same measurements were performed at $\nu = 5/3, 2, 8/3, 3$ yielding the expected values of $e^* = e/3, e, e/3, e$, thus increasing the confidence in the $\nu = 5/2$ results.

While the measurement of the quasiparticle charge confirms the paired nature of the state, it does not allow to discriminate between possible pairing scenarios, most notably the non-Abelian Moore–Read Pfaffian state as well as its particle-hole conjugate, the anti-Pfaffian (Levin et al., 2007; Lee et al., 2007), but also the Abelian states such as the (3,3,1) Halperin state or a strongly-paired modification of the Moore–Read state.

Therefore more information is needed to zero down on the best candidate or even to determine the nature of its quasiparticle statistics.

The second recommended paper (by I. Radu et al.) attempts to fill this gap by studying the tunnelling conductance across a QPC. A prediction of the edge theory is that in the weak tunnelling regime, the differential tunnelling conductance $G_t \propto T^{2(g-1)} F(g, e^* I_t R_{xy}/kT)$ (Wen, 1992). (g is the scaling dimension of the quasiparticle propagator and can be evaluated for all candidate theories. Unlike the charge, it is generally different for different paired states.) This enabled Radu and coworkers to extract the values of the quasiparticle charge e^* and the scaling dimension g by studying both the temperature and current dependence of G_t. The best fit gave the values of $g = 0.35$ and $e^* = 0.17e$. However, since the expected value of e^* is $e/4$, the two-parameter fit appears to give a slight edge to the non-Abelian anti-Pfaffian state ($g = 0.5$) over the Abelian (3,3,1) state ($g = 3/8$), but definitely not ruling the latter out. Curiously, the Moore–Read state (with $g = 1/4$) finishes third with a noticeable confidence gap.

More experimental data is needed to determine conclusively the exact nature of the $\nu = 5/2$ FQH state. However, these two papers give us the first evidence of pairing which has been long suspected. Especially exciting is the prospect of non-Abelian quasiparticle statistics, which appears consistent with the observations of Radu et al. More indisputable evidence will likely come from the interferometric experiments, and the devices used for measuring charge can be considered as a first step towards more complicated designs required for such experiments. Should the excitations in the $\nu = 5/2$ state indeed turn out to be non-Abelian, one might also consider the developments reported here as an important step toward manipulating these excitations. In particular, the degree of control of tunnelling through the QPC demonstrated in the paper by Radu and coworkers looks very promissing.

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