Singular Fermi-Liquids from Duality of Field Theories to Theories of Gravity

Emergent quantum criticality, Fermi surfaces, and AdS2.

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arXiv:0907.2694; arXiv:0903.2477

Fermions and the AdS/CFT correspondence: quantum phase transitions and the emergent Fermi-liquid.

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arXiv:0904.1993

Recommended with a Commentary by Elihu Abrahams, Rutgers, Andrew Cohen, Boston University and Chandra Varma, University of California, Riverside.

An interesting recent development in theoretical physics seeks to find results of relevance to condensed-matter physics from the conjectured duality of supersymmetric conformal field theories (CFT) in a space of d (space plus time) dimensions to supersymmetric gravity theories in d+1 dimensional anti-deSitter space (AdS), the so-called AdS-CFT correspondence. A dictionary has been established for the correspondence (for a readable summary for the motivated non-specialist, see Horowitz and Polchinski [1]). The benefit is that usually insoluble field theory at strong coupling has a weak-coupling gravity theory as its dual. Asymptotic low-energy results for the field theory are obtained at the boundary of the extra dimension of the gravity theory.

Perturbing the CFT through the introduction of temperature and a non-zero density corresponds to the introduction of a black hole on the gravity side, whose mass and charge are related to the temperature and chemical potential in the field theory. For energies low compared to the chemical potential, the field theory no longer appears conformally invariant. This makes the theory possibly relevant to condensed matter physics problems, which lack such invariance.

An important requirement for the correspondence is large N: the number of degrees of freedom in the CFT must be large. The leading results calculated on the gravity side then apply to the leading correlation functions in the CFT. In fact, the calculations are done in a double limit: $N \to \infty$ and $\lambda = gN^2$ held constant. Here g is the gauge coupling in CFT. For a calculable therory, the limit $\lambda \to \infty$ is taken. It is important to keep in mind that the large-N limit is described by a "mean field", in which all higher connected multipoint correlators vanish. With these limits, the gravity theory is classical and soluble, at least numerically. Calculating corrections is usually hard.

A conformally invariant (massless fields and Lorentz invariance) field theory is often specified by a set of primary operators along with some additional information. These operators always include an energy-momentum tensor $T^{\mu\nu}$, as well as currents associated with conserved charges J^{μ}. Generally there are other primary operators $\mathcal{O}_{q,\Delta}$ specified by their Lorentz quantum numbers, their charges q, and their conformal dimensions Δ. The duality dictionary specifies that each such operator has a corresponding field in the gravity theory: for the energy-momentum tensor this field is the metric $g_{\mu\nu}$; for the conserved current a gauge field A_{μ}; and for each of the remaining operators a matter field ψ. The charge and spin of ψ are simply related to the charge and spin of the corresponding operator in the CFT. The mass of the field ψ is fixed by the operator's conformal weight Δ in units of the AdS curvature. For the conserved quantities, J, the conformal weights are known, and correspond to vanishing mass in AdS.

Previously, it was shown [2] that hydrodynamic results for the conserved current correlations $\langle JJ\rangle(\mathbf{q},\omega)$ are correctly obtained by solving the corresponding classical gravitational equations and following the duality approach for large N,λ. In this limit, calculations in the gravity theory have essentially no free parameters, and the result is independent of the spectrum of primary operators.

The new development to which attention is drawn here is to consider a charged, spin-1/2 operator $\mathcal{O}$ at a non-zero chemical potential. The mass of the corresponding ψ field on the gravity side is a variable parameter corresponding to the conformal weight of the operator $\mathcal{O}$. In the large N,λ limit numerical results are obtained for the retarded propagator $\langle\mathcal{O}\mathcal{O}\rangle$. The MIT and Leiden papers differ somewhat in their technical aspects. The MIT group has also given an analytical transcription of their low-energy results which may be stated as follows: The correlator on the field-theory side is *non-analytic in the frequency dependence of its self energy, while it is analytic in its momentum dependence*: $\mathrm{Im}\,\Sigma(\omega)\propto\omega^{2\nu}$. ν depends on the conformal dimension Δ of $\mathcal{O}$, as well as details of the coupling of the vector potential (dual to J^{μ}) to gravity in the AdS space. For example, when $\nu = 1/2$, the singularity is the form found in the marginal Fermi-liquid description of the quantum-critical regime (optimal doping) of the cuprate superconductors. This also appears to be the form in the quantum-critical regime of the heavy fermions and possibly in the pnictides.

These intriguing developments raise several questions for theorists trained in the traditions of condensed matter physics: Usually, as in the phenomenological marginal Fermi-liquid description, singularities in the single-particle propagator are inherited from higher-order correlations which acquire singularities near critical points. Understanding the microscopic basis for such singularities usually requires knowing the possible broken symmetry at the critical point and calculating the correlations of the order parameter. Indeed for the Cuprates as well as the heavy-fermions and the pnictides, broken symmetries associated with the region in the phase diagram in which singular fermi-liquid properties are found have been discovered. Can such physics or equivalently the low energy Lagrangian and its critical points be specified by the AdS/CFT duality? The marginal Fermi liquid also implies certain singularities in the correlation of conserved currents, $\langle J^\mu J^\nu \rangle(\mathbf{q}, \omega)$, which lead to interesting singularities in measurable properties like the conductivity. To leading order, these are missing in the results from the gravity calculation. Is such information buried in the correction to the theory in higher orders in $1/N$? What is the physical content of the other correlations that appear through the AdS-CFt duality in the large N, λ limit? For example, there are the supersymmetric partners which appear at intermediate energy with quantum numbers which appear to have no possible physical meaning for condensed matter physics. There are problems in condensed matter physics, for example the Luttinger liquid in $1 + 1$ dimension which have Lorentz-invariant low-energy singularities in their single-particle propagators. These seem to be inaccessible to the dual gravity in the anti-deSitter metric. Is there a different metric where the dual theory remains Lorentz invariant? How many classes of critical theories and their gravity duals are there?

Further work will doubtless elucidate these issues. Perhaps the general mathematical edifice of AdS/CFT duality can only classify universality classes of quantum-critical singularities relevant to condensed matter physics without addressing questions of the physical origin of the singularities. That alone would be of considerable interest. At the present time, we should be excited that quantum critical fluctuations, which are unlike critical fluctuations in classical phase transitions, are also discovered starting from a very different view point in theoretical physics. We might also hope that the duality point of view may provide insight into the theory of quantum gravity and string theory from the known and understood results in condensed matter physics.

[1] Gary T. Horowitz, Joseph Polchinski, arXiv:gr-qc/0602037

[2] Christopher P. Herzog, Pavel Kovtun, Subir Sachdev, Dam Thanh Son, arXiv:hep-th/0701036