"Perfect" fluids in nuclear, atomic and condensed matter physics:

Shear viscosity of strongly coupled N=4 supersymmetric Yang-Mills plasma

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Viscosity in strongly interacting quantum field theories from black hole physics

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Theory of the Nernst effect near quantum phase transitions in condensed matter, and in dyonic black holes

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New physics of a strongly correlated material is often first revealed in transport experiments. Still, we have virtually no reliable tool to calculate transport coefficients in the limit where the mean free path is short and/or the charge carriers are not well defined quasiparticles. An elegant development in string theory led to the precise calculation of transport coefficients in strongly coupled field theories and has inspired new results for transport near the superfluid-insulator transition, in cold atom systems and for graphene.

Evidence for low-viscosity flow in high density quark matter, as seen in relativistic heavy ion collisions, motivated the investigation of the shear viscosity, η, in quantum many particle systems [see E. Shuryak Progr. in Particle and Nucl. Phys. **53,** 273 (2004)]. η measures the resistance of a fluid to establishing transverse velocity gradients. The traditional approach to calculate η is by solving the Boltzmann equation. A transport scattering rate τ^{-1} is determined perturbatively, yielding $\eta \simeq \varepsilon_{typ}\tau$ with characteristic energy density ε_{typ}. The approach is applicable at weak coupling or for diluted systems, where τ is long and the shear viscosity large. A very beautiful alternative approach was pioneered by Policastro *et al.*, and is based upon the duality between strongly coupled conformal field theory and weakly coupled gravity in extra dimensions. Policastro *et al.* find

$$\eta = \frac{\pi}{8} N^2 \hbar \left(\frac{k_B T}{\hbar c} \right)^3 \tag{1}$$

for the shear viscosity at infinite coupling of a $3+1$ dimensional field theory (the large N limit of a super-symmetric $SU(N)$ Yang-Mills theory that is argued to resemble QCD). c is the speed of light. In the language of quasiparticle scattering, the result implies a purely Planckian relaxation time

$$\hbar\tau^{-1} \simeq k_B T, \tag{2}$$

independent on the coupling constant.

To have a sharper measure of what precisely small or large viscosities mean, Kovtun *et al.* "normalized" η by the entropy density s. η has units of $\hbar\times$density, while s is of course measured in units of $k_B\times$density, making their ratio a dimensionless quantity times $\hbar/k_B$. The entropy density obtained in the same strong coupling limit is $s = \frac{\pi^2}{2}N^2 k_B\left(\frac{k_B T}{\hbar c}\right)^3$, as expected for massless fermions and bosons. Together, these two results imply that $\eta/s = (4\pi)^{-1}\hbar/k_B$ is a universal number. It was then postulated by Kovtun *et al.* that

$$\eta/s \geq \left.\eta/s\right|_c = \frac{1}{4\pi}\frac{\hbar}{k_B}, \tag{3}$$

for a large class of equilibrium systems. The equal sign refers to the above strong coupling limit. A fluid where η takes its lowest value is refereed to as a *perfect fluid*. These impressive results and the postulate, Eq.3, lead to a number of interesting questions for condensed matter physics systems:

What can one learn about quantum critical transport? The scale invariance of conformal theories makes them relevant for the description of critical points. The new aspect of the above listed theories is that they enable us to evaluate the corresponding scaling functions or universal coefficients close to a strong coupling fixed point, where transport is not due to the infrequent collision of well defined quasiparticles. In case of two dimensional systems this was done by Hartnoll *et al.*, who obtained exact results for the conductivity of a strongly-coupled conformal field theory and applied them to systems near a superfluid-insulator transition.

Does the above bound apply to all fluids? A counter example to the above bound was already mentioned in the original paper by Kovtun, Son, and Starinets: a classical fluid with large number of nonidentical species has an arbitrarily large mixing entropy without necessarily affecting the viscosity. It was however proposed that Eq.3 applies to non-relativistic single-component systems. All known condensed matter systems obey Eq.3. In classical fluids, the minimum value of η/s is taken at the critical point, with $\eta/s \gtrsim 10\left.\eta/s\right|_c$. In ^{4}He, a minimum value close to $9\left.\eta/s\right|_c$ is reached at the λ-point. Theories for cold atom systems with diverging scattering length and for clean graphene predict η/s to be smaller than for ^{4}He (but still larger than $\eta/s|_c$). While it seems that Eq.3 is not a rigorous bound for all equilibrium fluids, it clearly sets an order of magnitude limit. Only critical systems have η/s values that come close to $(4\pi)^{-1}\hbar/k_B$.

Are there similar bounds for other transport coefficients? Boltzmann theory of massless fermions or bosons gives $\eta/s \simeq \hbar/k_B \times l_{mfp}/\lambda_T$, where l_{mfp} is the mean free path and λ_T the mean distance of thermally excited particles. A saturation of η/s at $\hbar/k_B$ times a number of order unity corresponds the saturation of l_{mfp} at the mean carrier distance. In case of the electric conductivity this corresponds to the Mott-Ioffe-Regel (MIR) limit. Despite signs for a saturation of the resistivity in some systems, evidence for the violation of the MIR limit in high temperature superconductors at elevated temperatures is an important clue that no rigorous bound for the conductivity exist. Of course, this does not imply that Eq.3 for η is violated in the cuprates.

Can one come up with a tighter bound in Fermi liquids? For a Fermi liquid, η/s is not of order unity even if one sets $k_F l_{mfp} = 1$. Within Boltzmann theory follows $\eta/s \simeq g^{-2}\frac{\hbar}{k_B}\left(E_F/\left(k_B T\right)\right)^3$ with dimensionless coupling constant g. η/s is large, even if $g \sim \mathcal{O}(1)$. However, comparing the diffusive shear mode of a relativistic fluid (graphene, the quark gluon plasma etc.) with that of a Galilei invariant fluid (of velocity v and mass m), suggests the "normalization" $\eta T/\left(mv^2\rho\right) \simeq g^{-2}E_F/\left(k_B T\right)$, instead of η/s, as candidate bound for $g \sim \mathcal{O}(1)$.